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PATTERN CONFINEMENT AND THE HIGGINS-LEIGHTON METHOD

by



A. JAIN

A THESIS

SUBMITTED TO THE FACULTY OF GRADUATE STUDIES  
IN PARTIAL FULFILMENT OF THE REQUIREMENTS FOR THE DEGREE  
OF MASTER OF SCIENCE IN PETROLEUM ENGINEERING

DEPARTMENT OF CHEMICAL AND PETROLEUM ENGINEERING

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The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies for acceptance, a thesis entitled "Pattern Confinement and The Higgins-Leighton Method" submitted by A. Jain in partial fulfilment of the requirements for the degree of Master of Science in Petroleum Engineering.



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## ABSTRACT

This study consists of two parts. The first part deals with a laboratory study of pattern flooding. A series of experiments was conducted to investigate the effect of back pressure and pattern confinement on oil recovery and areal sweep efficiency.

To study the effect of back pressure a normal five spot pattern was selected. A series of back pressures ranging from 3 cms. to 70 cms. of water column was studied. The results obtained indicate that oil recovery and areal sweep efficiency are independent of back pressure within the range of back pressures investigated.

In order to study the effect of pattern confinement, two patterns: a direct line drive, and a normal five spot, were selected. The confinement of a pattern was achieved by surrounding it with identical patterns around its periphery in an attempt to establish isopotential boundaries around the pattern of interest.

The direct line drive pattern was surrounded by two rows on either side whereas the five spot pattern was surrounded by eight five spots. The oil recoveries obtained in these two cases ranged from 88.2% for a direct line drive to 171.6% in case of the five spot.

The second part deals with an examination of the Higgins-Leighton method for predicting the performance of



a water flood in an oil field. Since this method is limited to the case where a constant pressure drop is maintained between the injection and production well, it was modified and extended to cover the following cases:

- constant injection rate
- varying injection rate and pressure drop.

The original and the modified Higgins-Leighton methods were compared with a laboratory study of a five spot pattern reported in the literature. The agreement between the experimental and the calculated results was found to be better in the case of the modified approach.



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P A R T      O N E

THE EFFECT OF PATTERN CONFINEMENT

ON OIL RECOVERY AND AREAL SWEEP

EFFICIENCY



## INTRODUCTION

A full utilization of pilot test results and reliable prediction of large scale water floods requires a knowledge of the relationship between the recovery performance of confined and unconfined well patterns. It was the objective of this investigation to examine the effect of pattern confinement on oil recovery and areal sweep efficiency in the case of a five spot pattern.

To date only limited basic information is available on the behavior of confined pilot water floods. Dalton, Rapoport and Carpenter (1) investigated the effect of the ratio of pressure drawdown at the producing well to the pressure build up at the injection well ( $\pi$ - ratio), on oil recovery in confined and unconfined pattern floods. Caudle and Loncaric (2) on the other hand studied the effects of changing the ratio of injection rate to withdrawal rate on oil recovery for confined and unconfined five spot pilot floods. However, hardly any of the work to date has dealt explicitly with oil recovery and areal sweep performance of confined pilot floods.

A considerable amount of experimental work in this direction has been conducted at the University of Alberta. This work was reviewed. It was discovered that the earlier workers had difficulties with regard to model expansion, partial well penetration and cleaning of the model. Since a



larger number of wells was needed to achieve confinement in the present work and the problems encountered in the previous work were to be avoided, it was decided to construct a new model.

Finally, it was decided to reinvestigate the effect of back pressure on oil recovery and areal sweep efficiency in order to resolve the controversy expressed by John W. Serra (3) and S.K. Bhatia (4).



### LITERATURE SURVEY

The recovery of oil from a pilot flood is governed by the following factors:

1. Amount of oil originally present in the reservoir.
2. Rate of water injection.
3. Viscosity of oil, injection water and connate water.
4. Mobility ratio  $M = \frac{\lambda_{\text{displacing}}}{\lambda_{\text{displaced}}} = \frac{\lambda_w}{\lambda_o} = \frac{k_w}{k_o} \frac{\mu_o}{\mu_w}$
5. The original free gas saturation.
6. Interfacial tension between oil and water.
7. Pore size distribution which together with interfacial tension affects the capillary pressure characteristics of the rock.
8. Wettability of the rock.
9. Gravity effects.
10. Pattern configuration.

A comprehensive literature review covering the effects of these parameters on oil recovery may be found in the work of Neilson (5), Pritchard (6), Culham (7), Serra (8) and Bhatia (9).

Neilson(10) investigated the effect of a pre-established free gas saturation on the sweep efficiency of an isolated inverted five spot and concluded that the ultimate recovery may be slightly increased by establishing a free gas saturation prior to water flooding. He also pointed out that the producing water-oil ratio for an



isolated pattern increases very slowly after breakthrough so that ultimate areal sweep efficiencies six times that at breakthrough are possible.

Culham (11) conducted a study of areal sweep efficiency as a function of water injected, for a wide range of mobility ratios, in the case of a nine spot pattern and concluded that an increase in mobility ratio causes the areal sweep efficiency and oil recovery to decrease. He also studied the effect of injection rate on the performance of a water-wet two dimensional system and was of the opinion that oil recovery decreases with increasing rate until stabilized conditions are obtained.

Pritchard's (12) work was concerned with obtaining more information on oil recovery after breakthrough in the case of an isolated nine spot pattern. During the course of his investigation, the effect of injection rate on oil recovery was also studied. The results obtained from the experimental work indicated that the critical rate at which oil recovery is rate independent did not exist for the system under study; instead an optimum rate was obtained such that the oil recovery decreased at injection rates higher than this optimum rate.

Serra (13) investigated the effect of back pressure on the production performance and sweep-out patterns for the case of a single normal five spot. It was concluded from the experimental results that the area contacted by



the displacing fluid is sharply reduced as the back pressure on the system is increased. He also pointed out that a considerable amount of production could come from the area beyond the well pattern; hence, a single isolated pattern may not give accurate estimates of the expected performance of a fully developed flood.

Bhatia (14) studied the effect of back pressure on oil recovery and areal sweep efficiency in the case of the isolated normal five spot and four inverted five spot patterns. It was concluded, on the basis of experimental results, that back pressure affects neither oil recovery nor areal sweep efficiency. Bhatia (15) was of the opinion that the effect of back pressure measured by Serra was due to the expansion in the experimental model.



### EQUIPMENT DESIGN

After reviewing the work of Serra (16) and Bhatia (17), it was decided to retain the major features of their design. That is, the model should consist of a 1/4 inch thick bead pack sandwiched between two lucite sheets. However, if the patterns were to be confined a greater number of wells would be required and, therefore, the size of the model would have to be increased. In addition; the wells were used to fasten the confining lucite sheets to prevent model bulging.

### DESIGN OF OILFIELD MODEL

The size of the model was dictated by the availability of material rather than engineering design. Since the only readily available thick lucite sheet was 4 feet square x 2 inches thick, it was decided to build the model of that material rather than to delay the project. Consequently, the design consisted of two, two inches thick lucite discs having a diameter of four feet.

A total of 16, 1/8 inch, holes was drilled on a circle of 3 feet 11 1/4 inches diameter to accodate the wells used for cleaning the model. In order to facilitate a study of various patterns, a total of 185, 1/8 inch, holes was drilled on a square pattern in the central area



of these sheets. These holes were drilled in both the sheets simultaneously to avoid misalignment problems while inserting the wells.

These two lucite sheets were separated by a 1/4 inch lucite washer having an outside diameter of 4 feet and an inside diameter of 3 feet 11 5/8 inches. This assembly was then fastened together around the periphery by means of 72, 1/4 inch, brass bolts. A seal was effected between the three lucite pieces by embedding a neoprene 'o' ring in each of the thick lucite discs opposite the inner washer.

A detailed drawing of the oilfield model design is presented in Figure I.

#### DESIGN OF OIL WELLS

The wells were designed so as to accomplish the following objectives:

- to provide a facility for injecting and withdrawing fluids from the model, and
- to fasten the lucite sheets together in such a manner as to avoid the model - expansion problem.

The wells consisted of two parts:

1. Well Head
2. Well Body



## OIL FIELD MODEL DESIGN

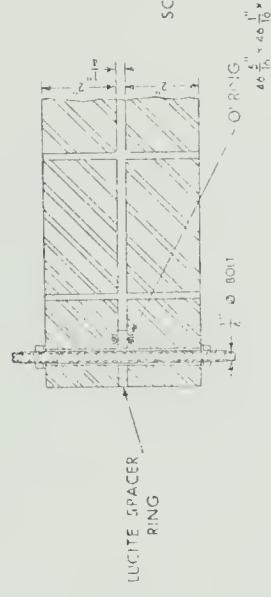
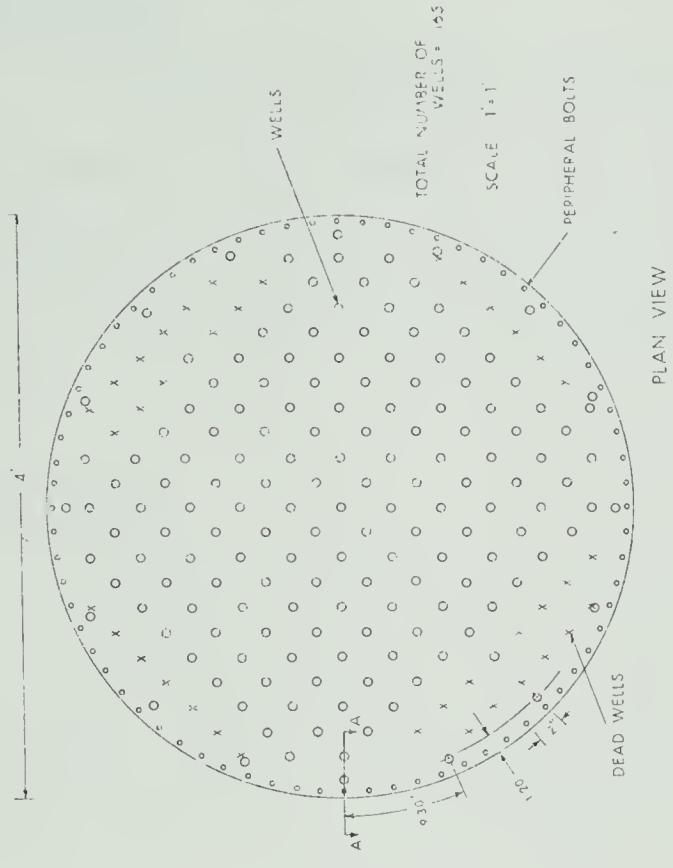
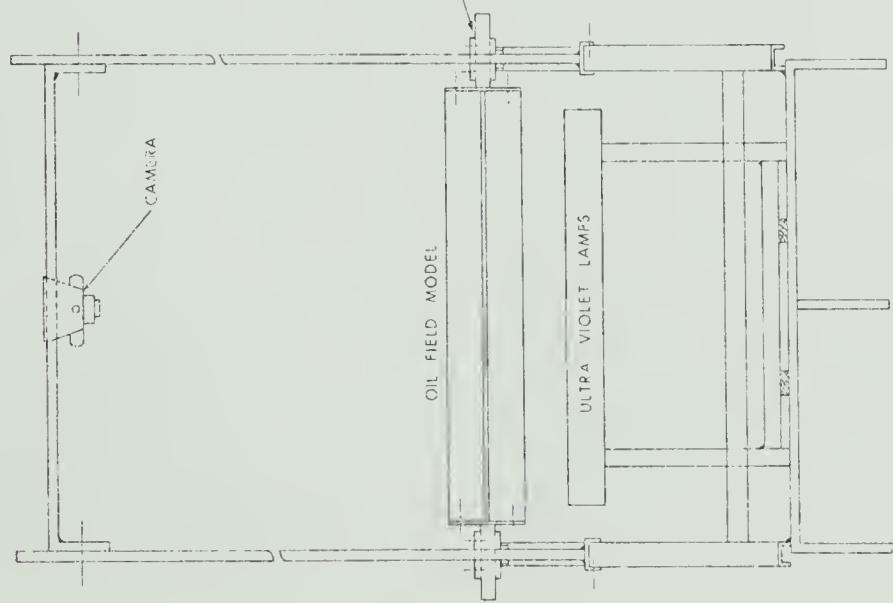


FIGURE - 1



1. Well Head

The well head was constructed from a 7/8 inch hexagonal brass stock. This piece of material was drilled on one end to accept a 1/8 inch N.P.T. valve and on the other end to accept a 1/8 inch diameter brass tube.

2. Well Body

The well body was constructed from a brass tube 1/8 inch in diameter and 2 1/4 inches long. One end of this tube was soldered into the well head and the other extended into the model through the bead pack. The section of this tube exposed within the pack was drilled with 4, 1/32 inch diameter holes, disposed on two different planes and situated around the circumference of the tube. A solid 1/8 inch brass rod was soldered onto the bottom of the tube. The rod extended through the lower lucite plate and its protruding end was provided with #5-40 threads.

The entire well was sealed with 'o' rings, one at the top, and another next to the nut and washer at the bottom. A detailed sectional drawing of a typical well is presented in Figure II.



## WELL DESIGN

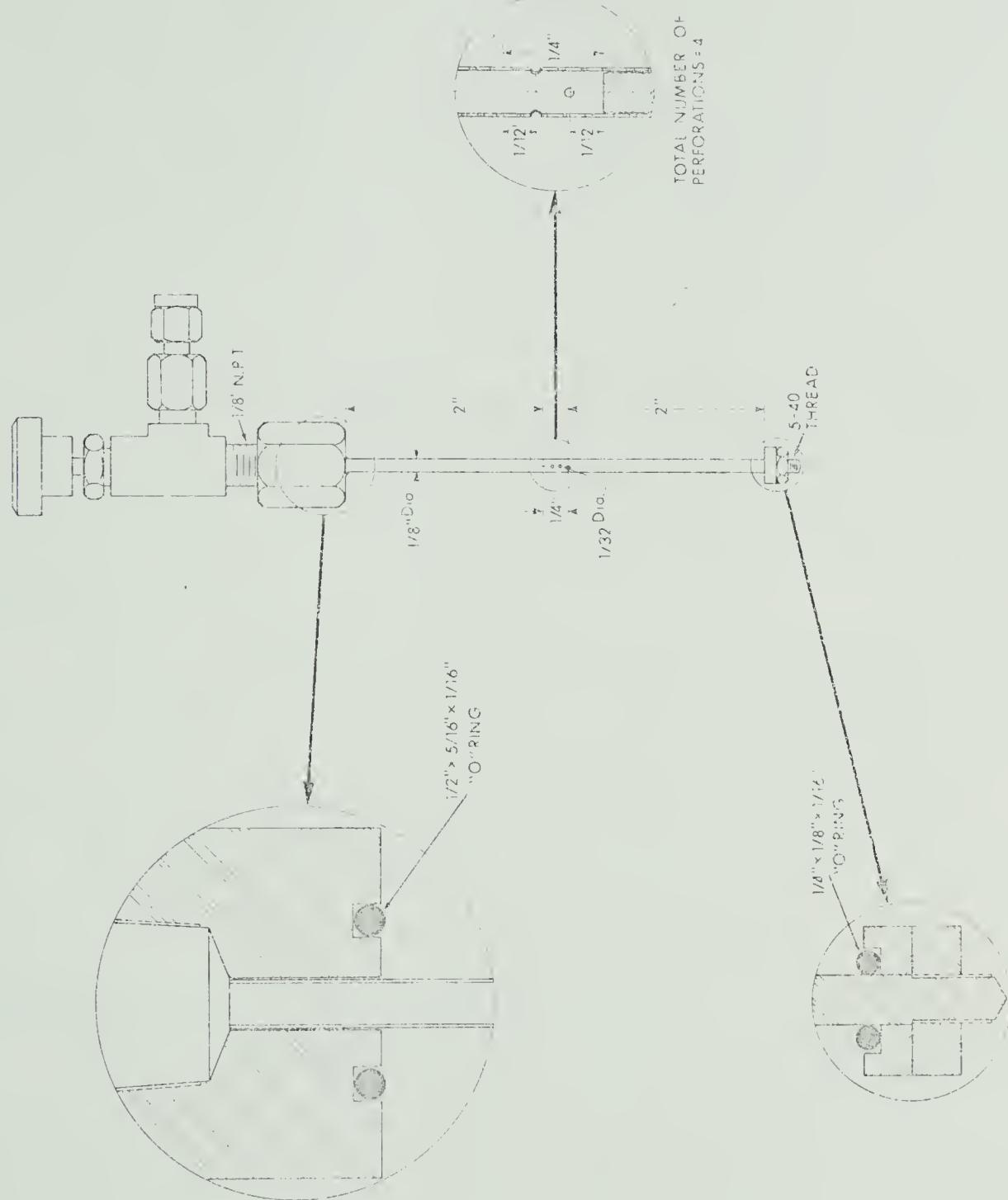


FIGURE - 11



## DESIGN OF INJECTION SYSTEM

In order to permit the confinement of a five spot pattern, it was felt that a minimum of sixteen injection lines would be required. The injection system was, therefore, designed to fulfill this condition. This was achieved by arranging seventeen stainless steel cylinders according to the pattern presented in Figure III. These cylinders were bolted to a bottom plate.

Of these seventeen cylinder-piston assemblies the one positioned in the center of the array was designed to be double acting. This design allowed it to be used to raise and lower the top plate, thereby driving the pistons in the surrounding cylinders in unison.

Since a constant injection rate was desired, the double acting master cylinder was driven by means of a Ruska proportioning pump (Cat. No. 2249, W II, Ser. No. 7681). This pump was capable of providing a total of 28 constant injection rates as presented in Table - I.

## PRESSURE TESTING OF THE OILFIELD MODEL

Before packing the model with glass beads, it was necessary to pressure test it in order to ensure that no leakage or well breakage would occur during the experimental runs. In order to conduct the pressure test, the wells were mounted on the model and held in position by tightening the nuts provided at their base. These nuts were tightened very carefully to avoid the development of an uneven stress



LAY OUT OF CYLINDERS IN - CYLINDER ASSEMBLY

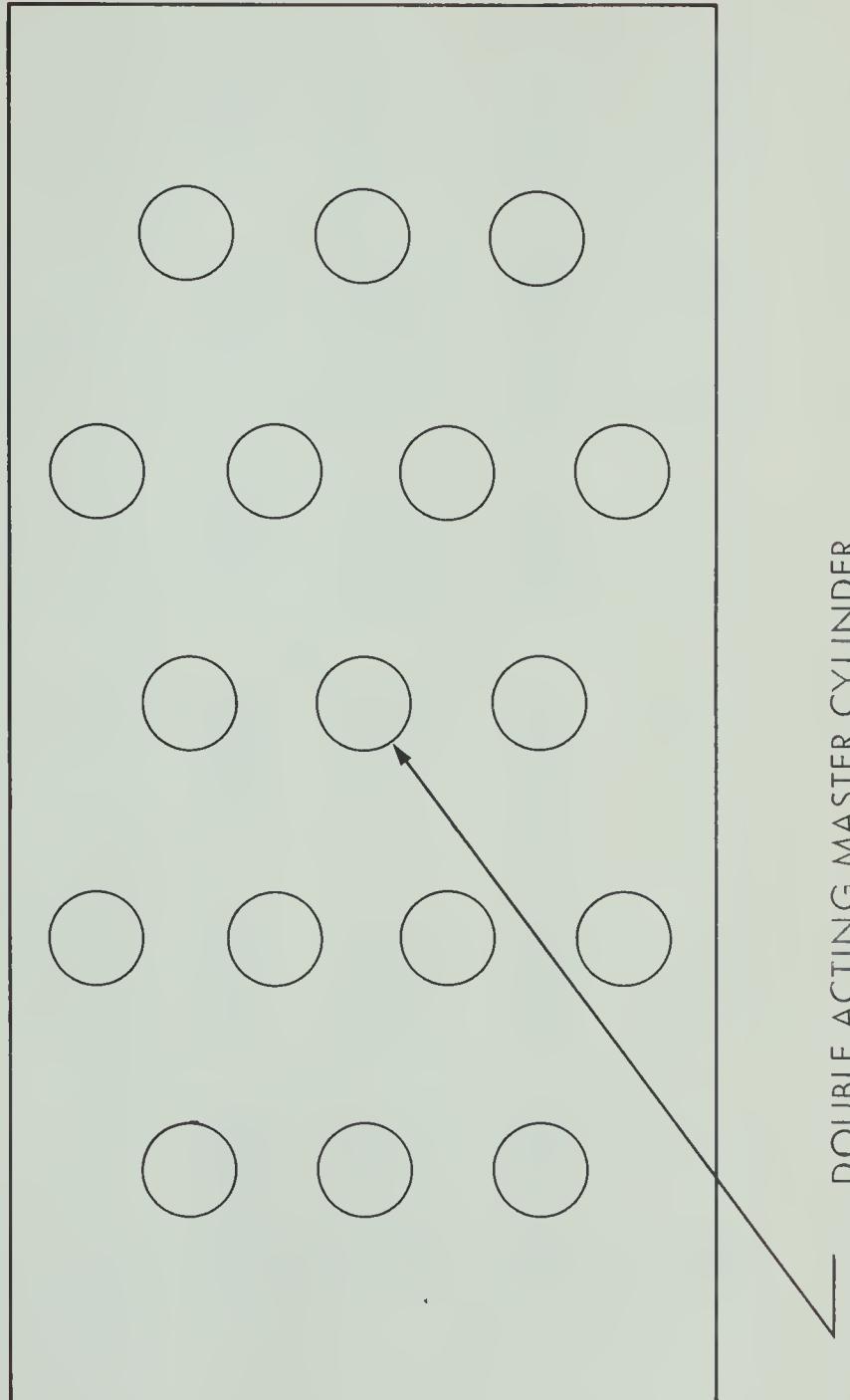


FIGURE - II



TABLE I

RUSKA PUMP - FEED RATES

LEVER POSITION	GEAR POSITION	FEED RATES	ccs/hr	7	6	5	4	3	2	1
A	2.500	3.125	3.750	5.000	6.250	7.500	8.750			
B	10.000	12.000	15.000	20.000	25.000	30.000	35.000			
C	40.000	50.000	60.000	80.000	100.000	120.000	140.000			
D	160.000	200.000	240.000	320.000	400.000	480.000	560.000			



distribution. The model was then completely filled with water and pressure tested using the Ruska pump mentioned earlier. The pressure in the model was observed on a manometer connected to one of the peripheral wells.

The test was conducted by raising the test pressure step-wise. If and when various well assemblies failed they were removed, repaired, and reinstalled. This procedure was continued until a test pressure of 9 p.s.i. could be maintained without leakage or mechanical failure. This pressure of 9 p.s.i. was selected on the basis of the following considerations:

1. Pressure drop due to flow in the old model.
2. Pressure due to a column of 70 cms. of water (maximum back pressure imposed on the model).
3. A safety factor of 2.0.

#### PACKING OF THE MODEL

U.S. 30-40 industrial glass beads were used to pack the model. The beads were first placed in chromic acid for 3 days, then washed with an aqueous solution of sodium hydroxide, and finally washed repeatedly with distilled water until the wash water was neutral to litmus paper. These beads were then dried and sieved to remove clusters.

The model was thoroughly cleaned and dried by passing air through it. It was then clamped in the vertical position. Two vibrators [Dynamite models 222 and 223, capacity 10 watts each, frequency 60 cycles/second] were con-



nected to the model. The beads were weighed and poured into the model in batches. Each batch was allowed to settle for 3-4 hours before the next one followed. Just before complete fill up, the model was tapped gently with a lucite hammer to further promote packing. The entire process took about seven days and resulted in a homogeneous, uniform pack.



## EXPERIMENTAL WORK

### GENERAL PROCEDURE

The model was cleaned and brought to an initial water saturation condition, using the peripheral wells. The water used for injection purposes was drawn into the cylinder assembly, from an overhead water reservoir through a system of quick opening toggle valves. The water injection rate was fixed by choosing the appropriate gear position on the Ruska pump.

The water injected into the pattern under study contained a fluorescent dye. The flood pattern was traced by causing the dyed water to fluoresce. This was accomplished by placing an ultra-violet light source underneath the model. A Pentacon-Six Camera was used to take photographs of the flood front from time to time. This camera used a tri-X 120 black and white film, a distance of 5.5 ft., lens opening between 2.8-4 and exposure time of 5 seconds.

The injection pressure was measured using mercury manometers with a short and a long column of dyed water. The water containing the dye was made to fluoresce by two ultra-violet tubes mounted on an iron stand and facing the manometer board. These manometers were photographed using a Rolleicord Camera, with tri-X 120 black and white film, aperture between 2.8-4, and an exposure time of 10 seconds. A series of photographs was taken during each experimental



run. The pressure drops were then estimated by measuring the difference between colored limbs, using a travelling microscope. A schematic diagram of the equipment is presented in Figure IV. Figure V presents a photograph of the equipment, showing the layout of various units.

#### PRELIMINARY EXPERIMENTS ON THE MODEL

A number of preliminary experiments was conducted on the model to determine the porosity and absolute permeability of the pack. The calculation procedure used and the results obtained are presented in Appendix A.

#### POROSITY

In determining porosity, the bulk volume of the model was determined by carefully measuring the amount of water necessary to fill the model, prior to packing. Knowing the weight of the beads contained in the pack and their specific gravity, it was possible to calculate the matrix volume of the model. The bulk volume and the matrix volume were then used to calculate porosity. Alternatively pore volume was determined by measuring the volume of water required to completely saturate the packed model. Both methods yielded the same value for porosity, namely 36.43%.

#### ABSOLUTE PERMEABILITY

Since the model was completely saturated with water, a five spot pattern was selected and water injected into the model through the four injectors. The pressure drop due to flow of water was measured and the absolute permea-



## FLOW CHART

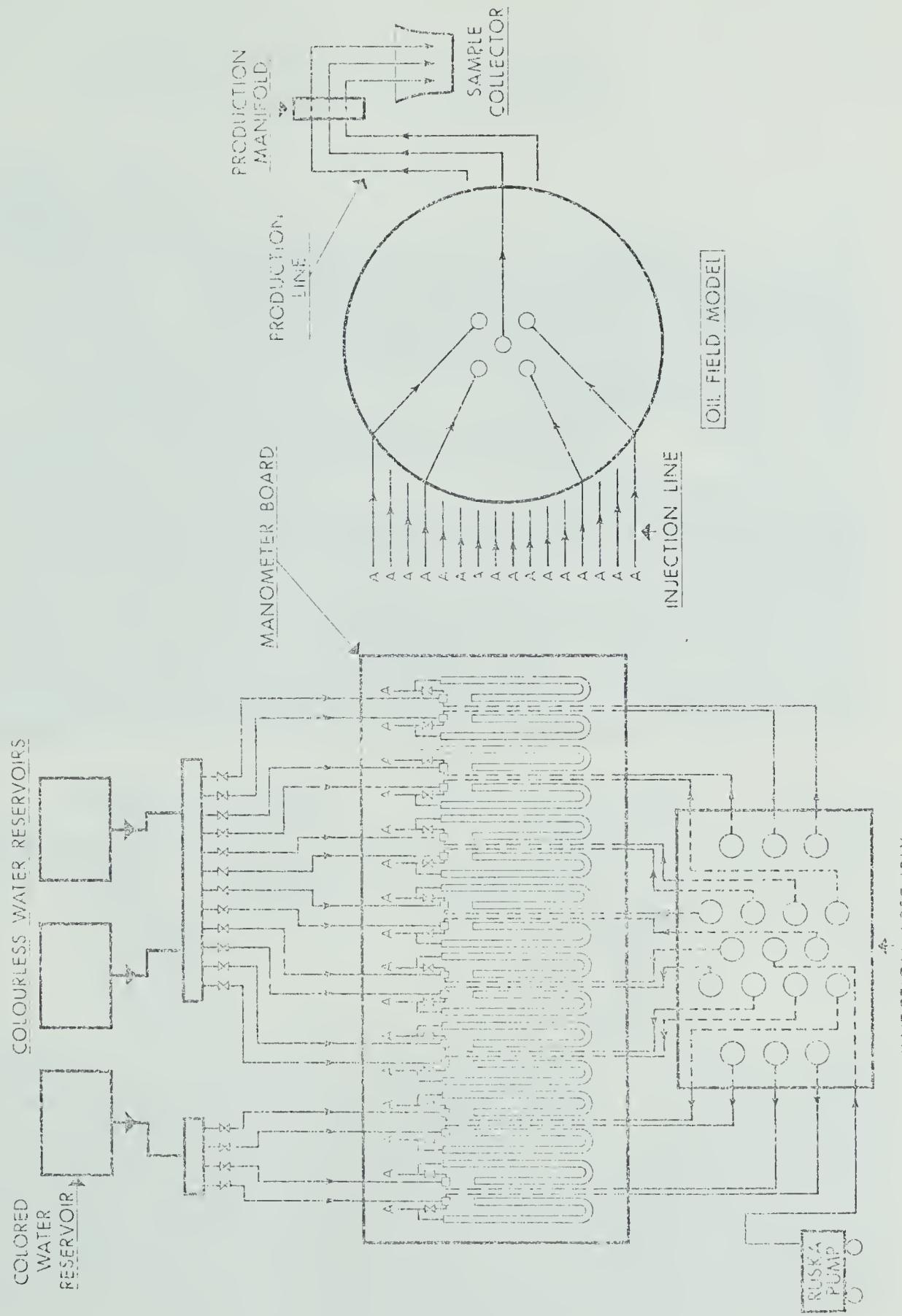


FIGURE V



PHOTOGRAPHS OF THE EQUIPMENT  
SHOWING LAYOUT OF VARIOUS UNITS

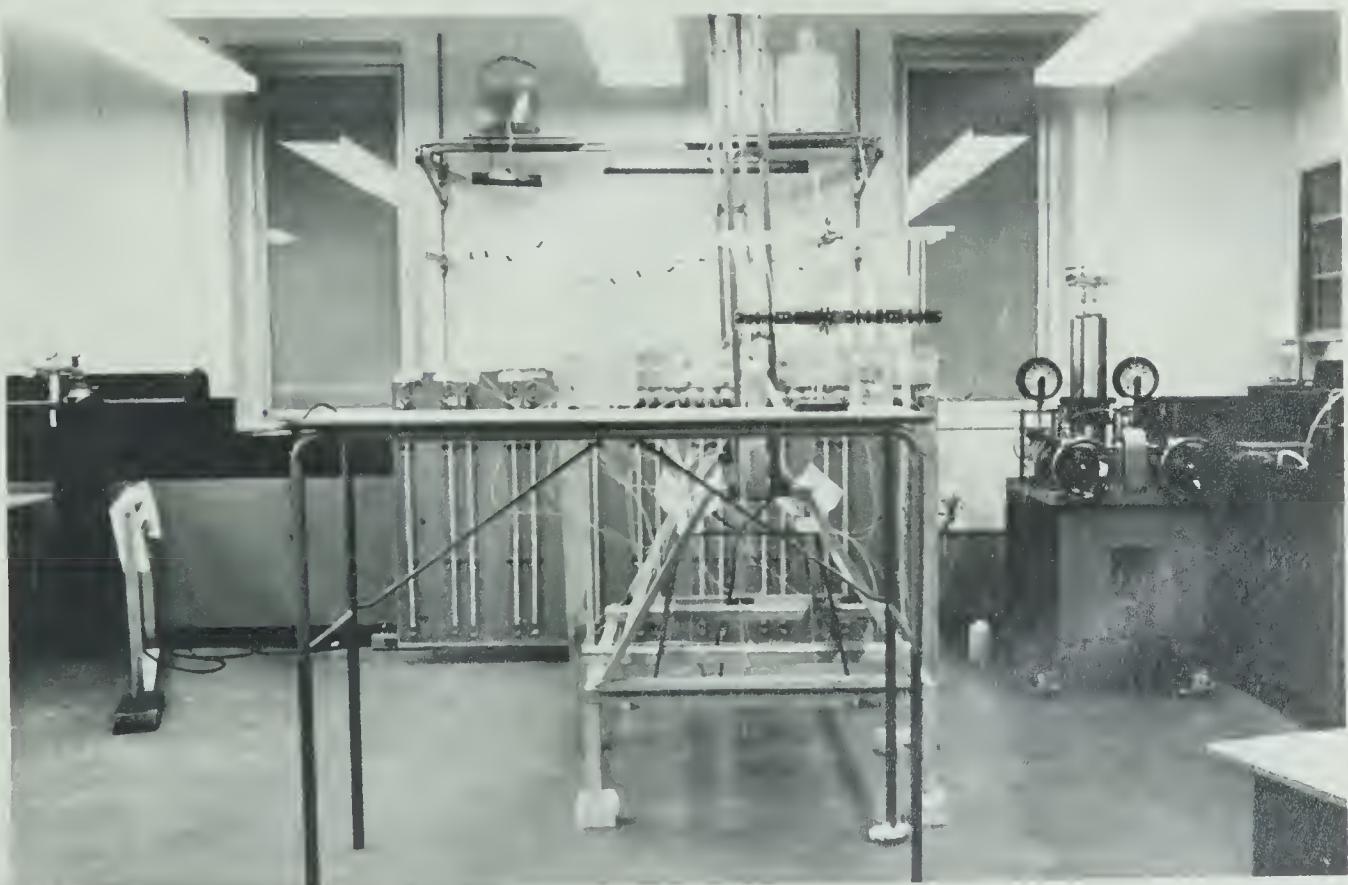


FIGURE - V



bility of the model calculated using Muskat's (18) steady state, radial equation for a five spot pattern. The absolute permeability of the model was found to be 27.7 darcys.

#### FLUID PROPERTIES

In the present investigation only two phases viz., oil and water were considered. The following properties for these fluids were determined.

1. Density - using a specific gravity bottle.
2. Viscosity - using a Cannon - Fenske Ostwald Viscometer.
3. Interfacial Tension - using a Dü-Nuoy Tensiometer.

The values obtained at a temperature of 78°F are presented in Table II.

TABLE II  
FLUID PROPERTIES AT 78°F

<u>FLUID</u>	<u>DENSITY</u>	<u>VISCOSITY</u>	<u>WATER-OIL INTERFACIAL TENSION</u>
Kerosene (Esso, Regu- lar grade)	0.7297	1.311	--
Distilled Water	0.9900	0.9680	32.140
Dyed Water	0.9908	0.9700	--

Injection water used to displace the oil was deaerated distilled water colored with a fluorescent dye (Sodium Fluorescein , Uranine B, Fischer Scientific). This dye



was completely soluble in the water phase but insoluble in the oil phase. A concentrate of the dye was made with 5 gms. of dye per 500 c.cs. of distilled water and 250 c.cs. of such concentrate was mixed in 8.25 litres of distilled water for injection purposes.

#### SCALING

The model was scaled so that the performance was not rate sensitive. This was accomplished by using the Rapoport, Carpenter and Leas (19) scaling coefficient as the criterion.

The critical injection rate was calculated for the system under study and the injection rates used in various experimental runs were kept above this critical value. The calculations for the critical injection rate are presented in Appendix B.

#### PATTERN CONFIGURATION

The following pattern configurations were selected for conducting the experimental runs.

1. Isolated Normal Five Spot.
2. Confined Normal Five Spot.
3. Confined Direct Line Drive.

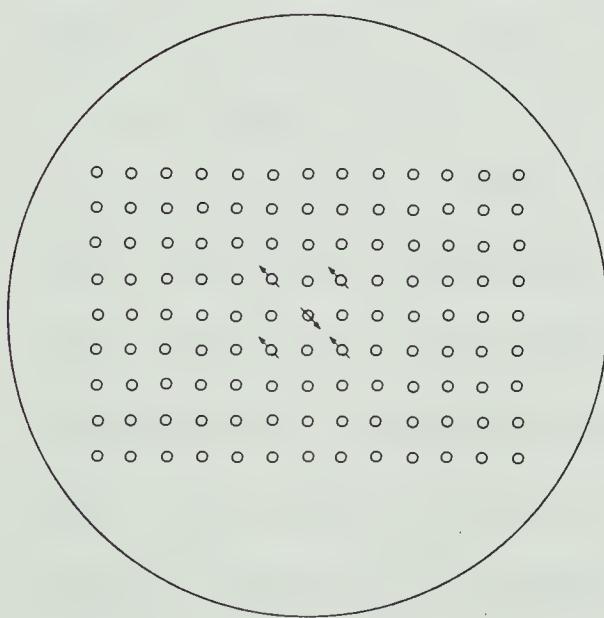
The arrangement of injectors and producers in each of these patterns is presented in Figure VI.

#### EXPERIMENTAL PROCEDURE

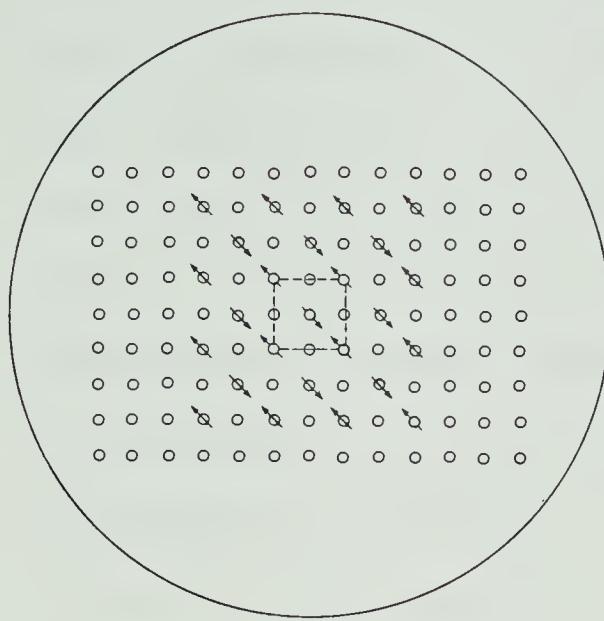
The experimental runs were carried out using the following steps:



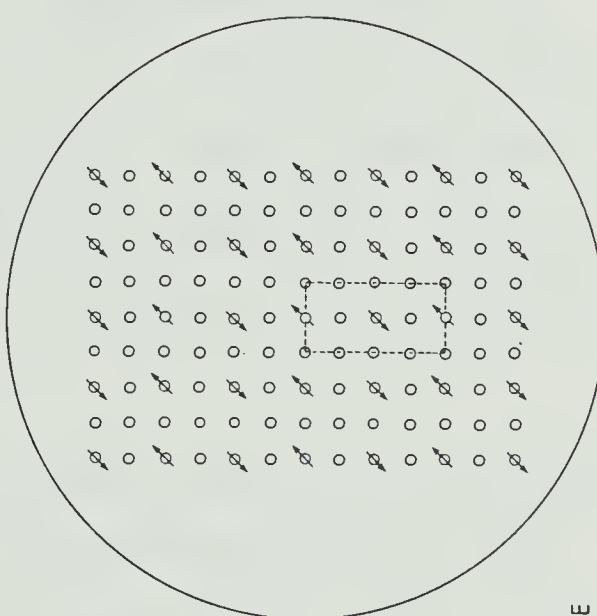
ARRANGEMENT OF WELLS IN VARIOUS PATTERNS STUDIED



ISOLATED NORMAL  
FIVE - SPOT PATTERN



CONFINED FIVE - SPOT PATTERN



CONFINED DIRECT LINE DRIVE



FIGURE — VI



1. In order to establish connate water saturation, the model was saturated with water and clamped in the vertical position. The oil was injected into the model through the two uppermost peripheral wells, while water was removed from one of the lowermost wells. The water produced was collected in a vessel kept at a height higher than the injection point in the model. This procedure helped in taking full advantage of the density difference between oil and water, thus maintaining the displacing front as nearly horizontal as possible. The injection of the oil was continued until no more water was produced. The model was then clamped in the horizontal position.
2. The flood pattern was selected and water was removed from the injection and the production well stems by injecting oil into the model through one of the peripheral wells. The production lines were filled with oil before starting the run.
3. Back pressure on the model was fixed by raising or lowering the production manifold to the desired level. The elevation of the manifold above the center of the bead pack was measured by means of a cathetometer.
4. The injection and the production wells were opened and sufficient time was allowed for the system



to attain pressure equilibrium. The initial pressure drop in the manometer was recorded. The gear position on the Ruska pump was adjusted to deliver the desired injection rate.

5. The pump was started and the production from the model was collected in a tube bank arranged underneath the production manifold.
6. The flood front was traced from time to time, by photographing the model. The volumes of fluids produced at each instant that the model was photographed were noted. The experimental run was continued until boundary effects tended to distort pattern growth.
7. After the run was over, the model was clamped in the vertical position. The five lowermost peripheral wells were connected to the injection lines and water was delivered to these lines from the overhead water reservoirs. The oil was removed from the model using the three uppermost peripheral wells and was collected. This process of flushing the model with water was continued until no more oil was produced and the model was completely free of dye.
8. Steps 1 through 7 were repeated for the next run.



The photographs obtained during the run were planimetered to find the area of the porous medium contacted by the flooding water at any stage. This allowed the calculation of areal sweep efficiency.



## EXPERIMENTAL RESULTS AND DISCUSSION

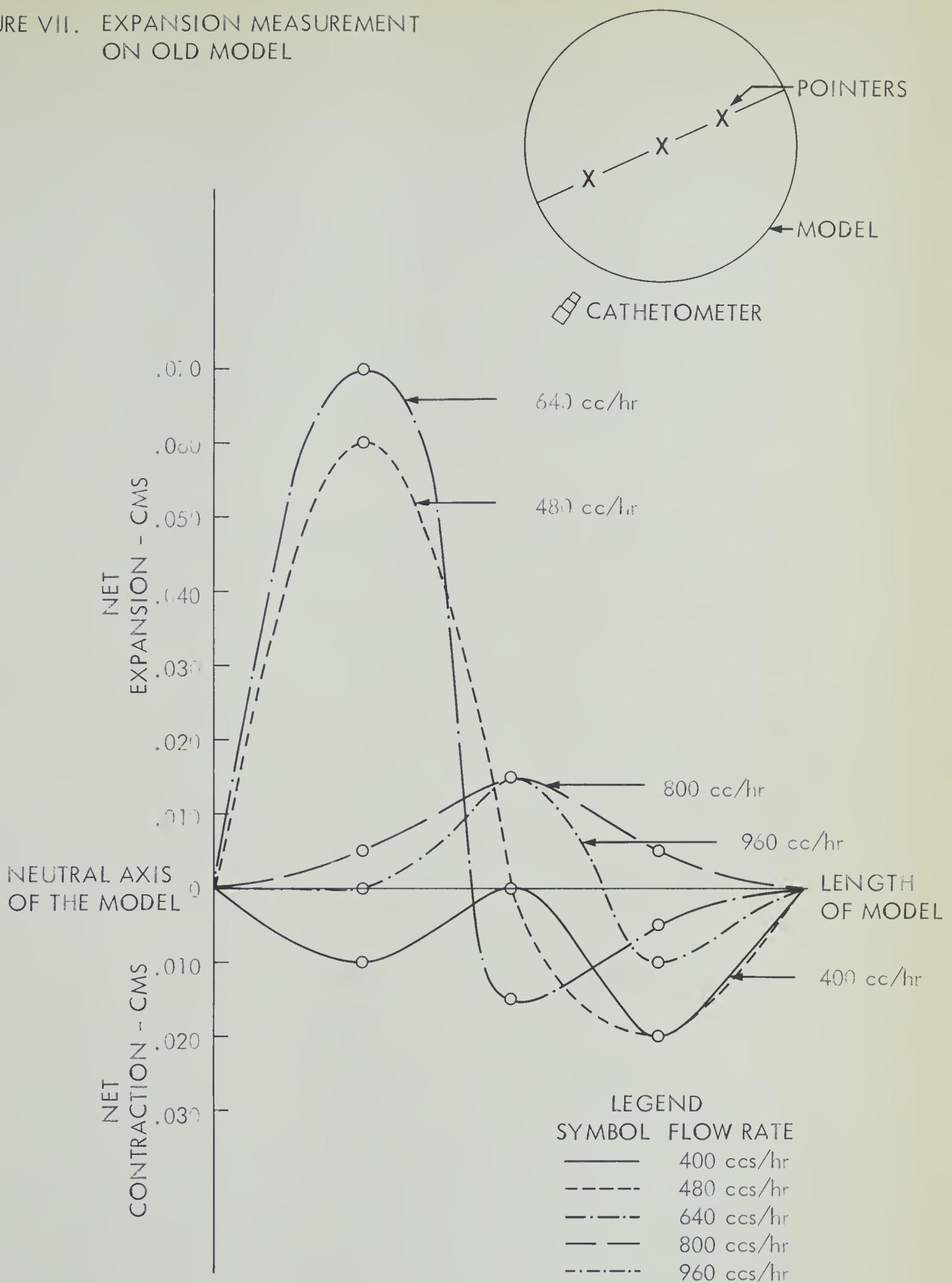
### MODEL EXPANSION STUDY

A preliminary study was conducted on Bhatia's model to determine the validity of his contention (20) that the model expanded during the course of experimentation. To do this, the model was saturated with water and three pointers were attached to the wells along a diameter on the top surface of the model. Similarly, three pointers were attached to the bottom surface of the model, directly underneath the pointers on the top surface. The injection lines were then connected to the model in the same five spot pattern, as used by Bhatia and Serra. Prior to starting the run, the initial elevations of the pointers were measured using a cathetometer. The injection rate was varied from 400 ccs/hr to 960 ccs/hr in steps and at each step, the elevation of the pointers was recorded. The results obtained are tabulated in Table III, Appendix C.

The net expansion-contraction curve shown in Figure VII indicates that the net expansion in the model increases with the injection rate up to a rate of 640 ccs/hr, beyond which point it decreases. It is interesting to note that one part of the model expands while the other contracts. This can perhaps be explained on the basis of irregular stress distribution in the model. A similar experiment was conducted on the new model and no measureable expansion



FIGURE VII. EXPANSION MEASUREMENT  
ON OLD MODEL





or contraction could be detected. Another experiment in this direction measured the increase in pore volume at various back pressures. The results of this experiment are shown in Figure VIII and tabulated in Table IV, Appendix C. It may be observed that 3.05 c.cs of oil was injected into the model at a back pressure of 70.0 cms. of water. This represents a net increase of 0.0003 cms. in the thickness of the model and is beyond the accuracy of the cathetometer.

An alternative approach to measure expansion in the model is by measuring pressure drop at various injection rates, with back pressure as a parameter. A radial flow experiment was set up for this purpose. The water was injected into the model from the peripheral wells and the production obtained through the central well. The results of this experiment are tabulated in Table V, Appendix C, and plotted in Figure IX. The linear relationship between pressure drop and flow rate indicates:

- that laminar flow persists in the porous medium at the flow rates studied,
- that there is no expansion in the model.

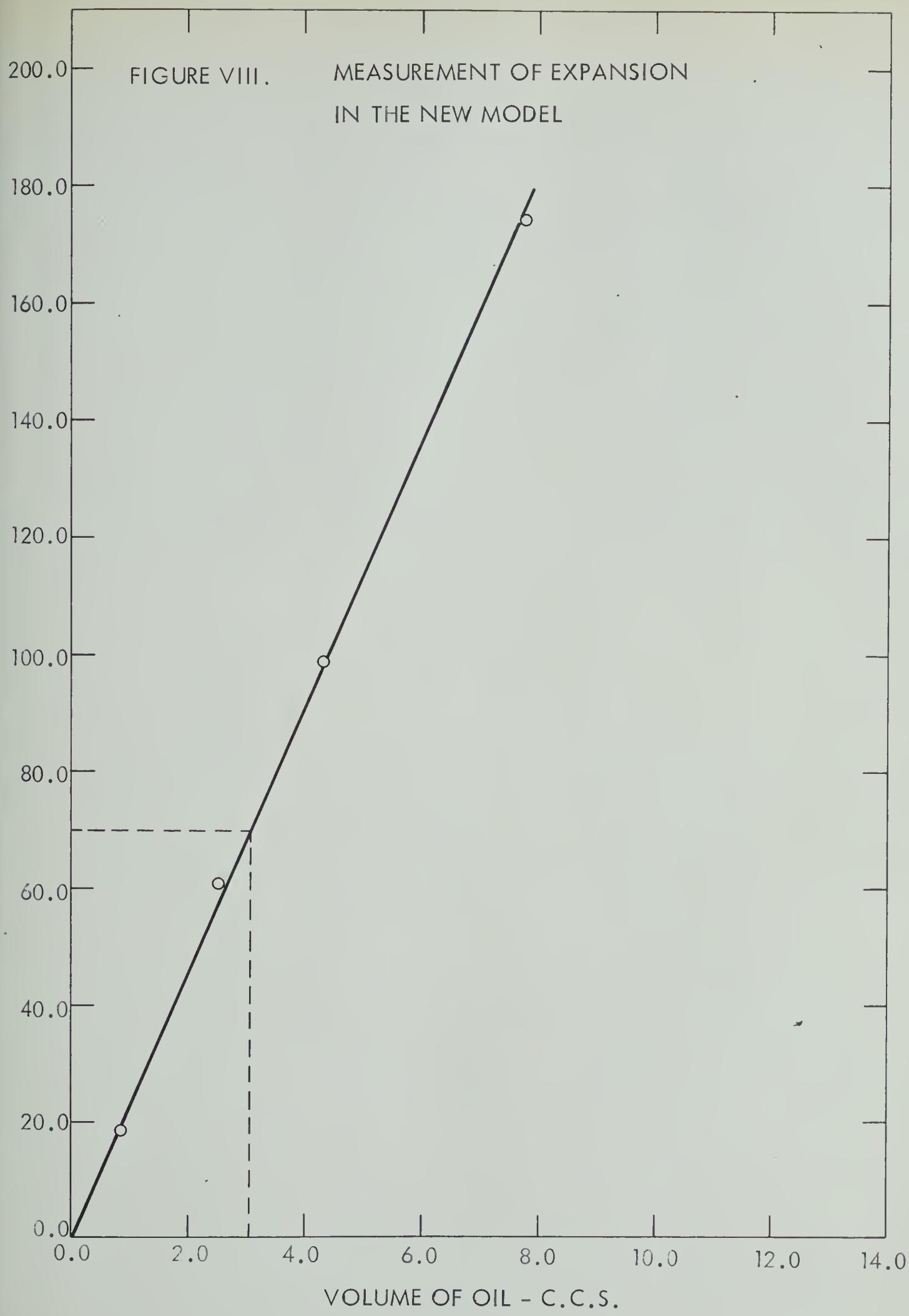
During the radial flow experiment, colored water was injected into the four peripheral wells located on the principal axes of the model. Figure X presents a pictorial representation of the pattern development from start to the completely developed stage. It was concluded from



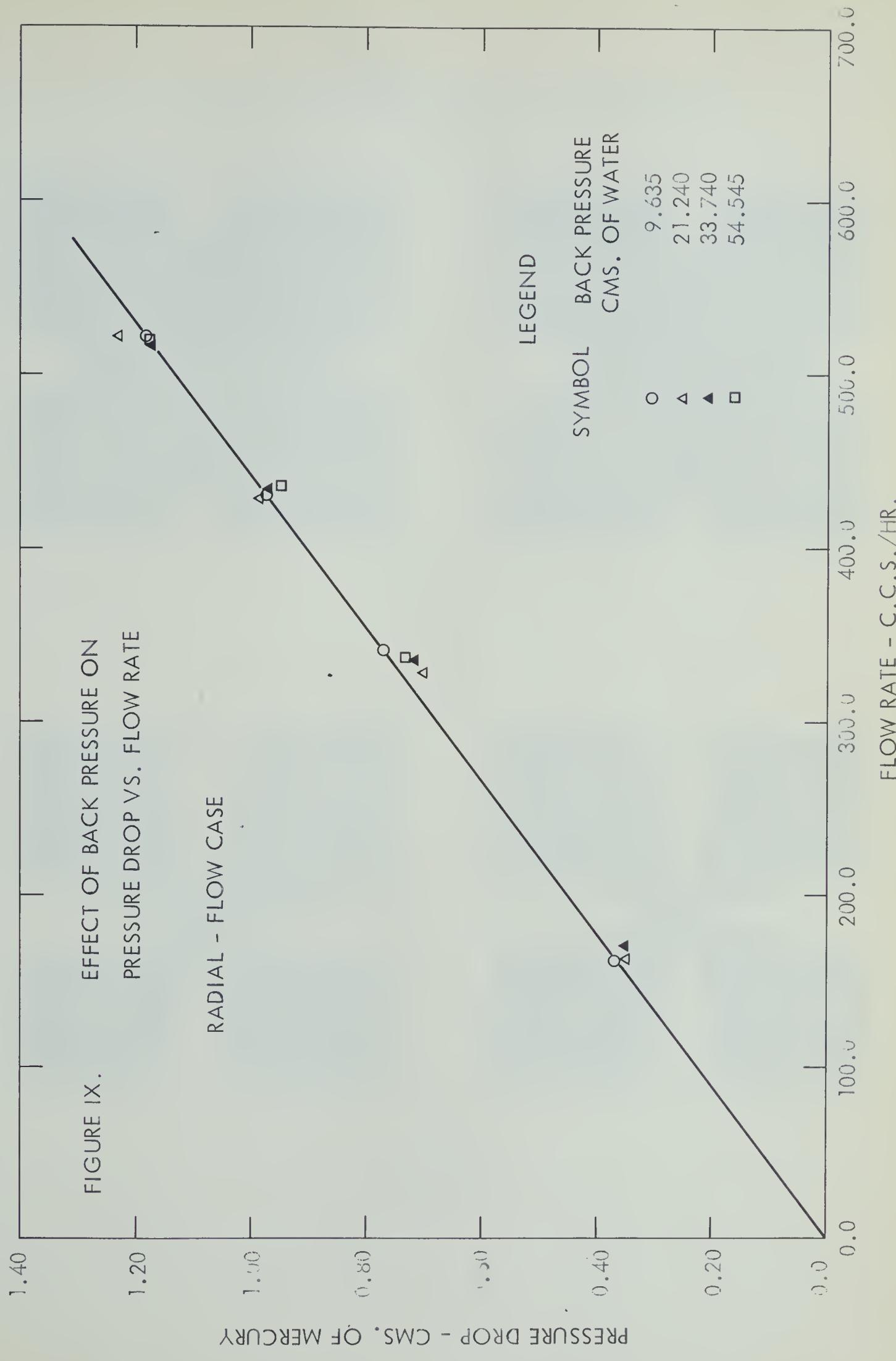
FIGURE VIII.

MEASUREMENT OF EXPANSION  
IN THE NEW MODEL

PRESSURE - CMS. OF WATER









PATTERN GROWTH IN RADIAL FLOW EXPERIMENT

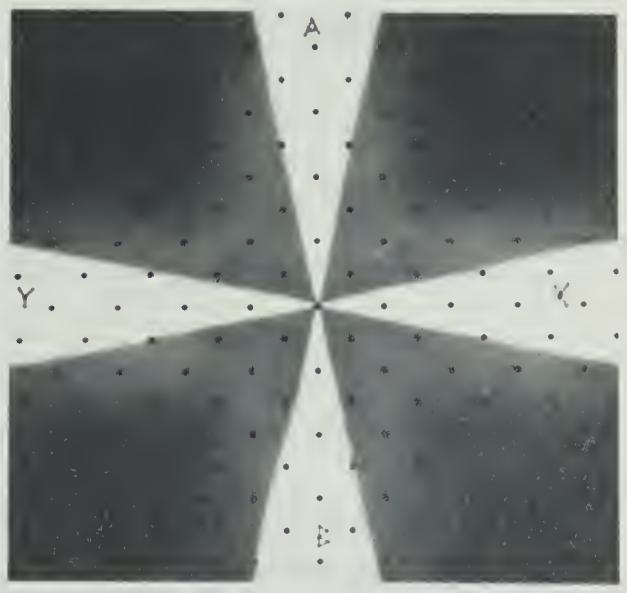
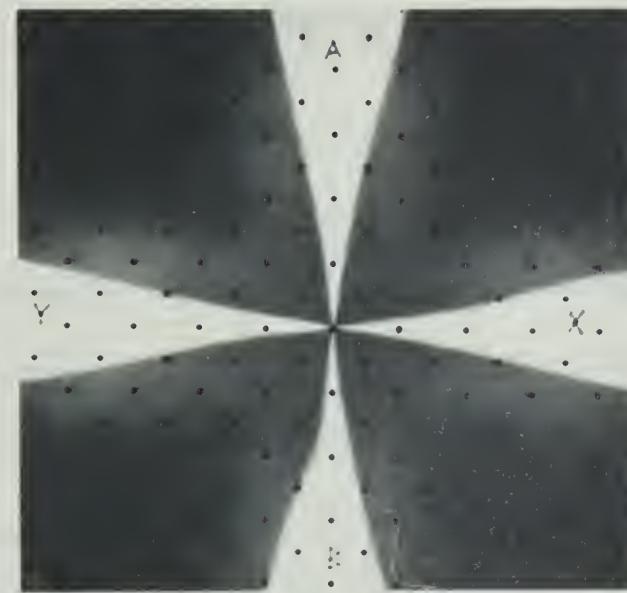
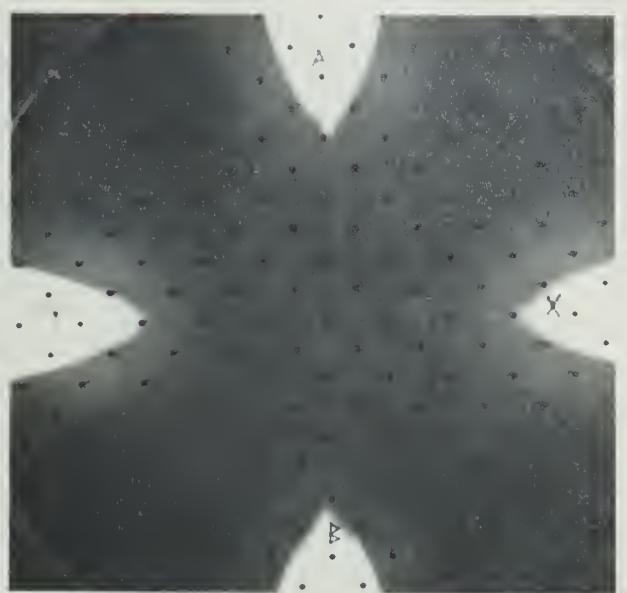
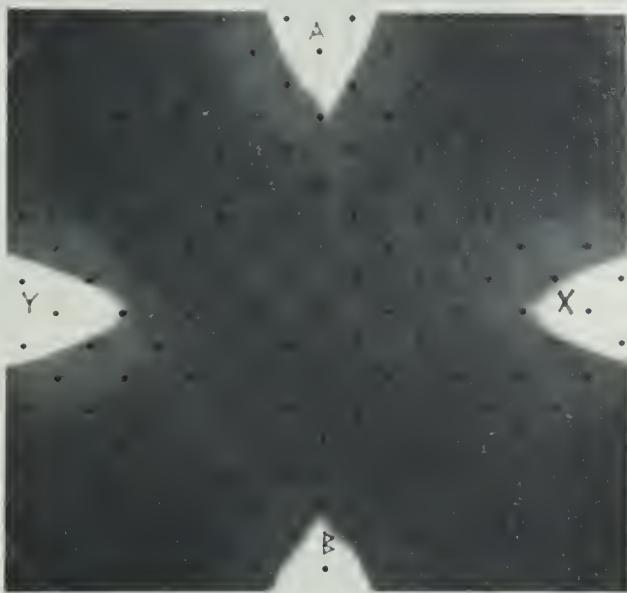


FIGURE X



these photographs that the packing in the model was uniform and homogeneous.

#### EFFECT OF BACK PRESSURE

As mentioned earlier, the oil recovery and areal sweep efficiency of a pattern flood is influenced by the following factors.

- 1) Gravity effects.
- 2) Capillary effects.
- 3) Injection rate.
- 4) Viscosity of oil, connate water and injection water.
- 5) Wettability of the porous system.
- 6) Pattern configuration.
- 7) Mobility ratio.
- 8) Interfacial tension between oil and water.
- 9) Amount of oil originally present in the reservoir.

Therefore, in order to investigate the effect of back pressure, it was important to ensure that all the factors listed above remained constant in the experimental runs. Since the thickness of the porous medium used to carry out this investigation was 1/4 inch, it could be safely assumed that gravitational effects were negligible. The effects due to capillarity and rate were held constant by selecting an injection rate higher than the critical, calculated from the scaling coefficient suggested by Rapoport, Carpenter, and Leas (21). Parameters



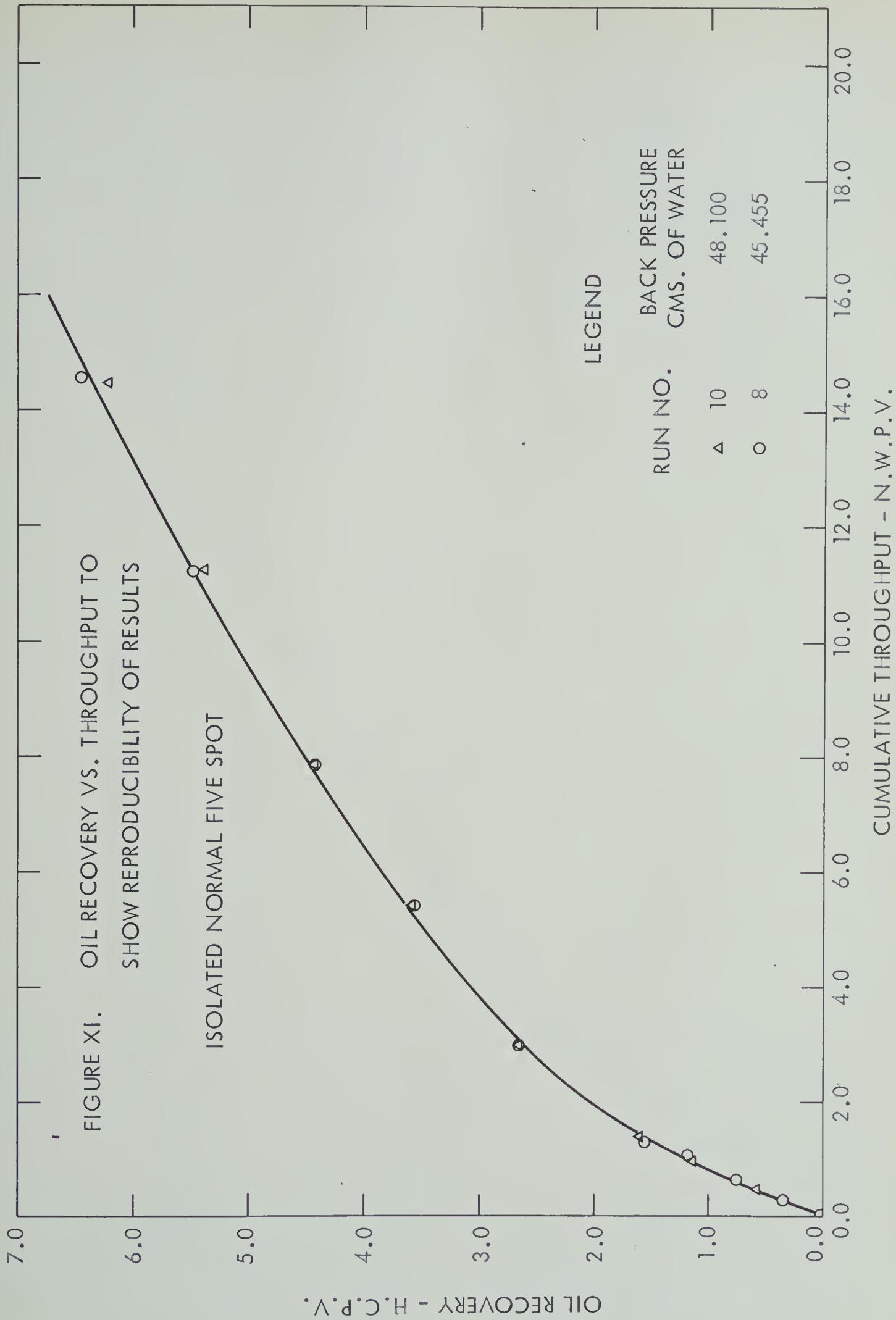
4-8 were held constant by using the same oil, water and pattern configuration in the various experimental runs.

The effect of back pressure was investigated by conducting a series of experimental runs (Runs 5-10). An isolated five spot pattern was selected and the results obtained are tabulated in Tables VI - XI, Appendix C. The back pressures studied, ranged from 3.0 to 70.0 cms. of water. Due to physical limitations, higher back pressures could not be considered.

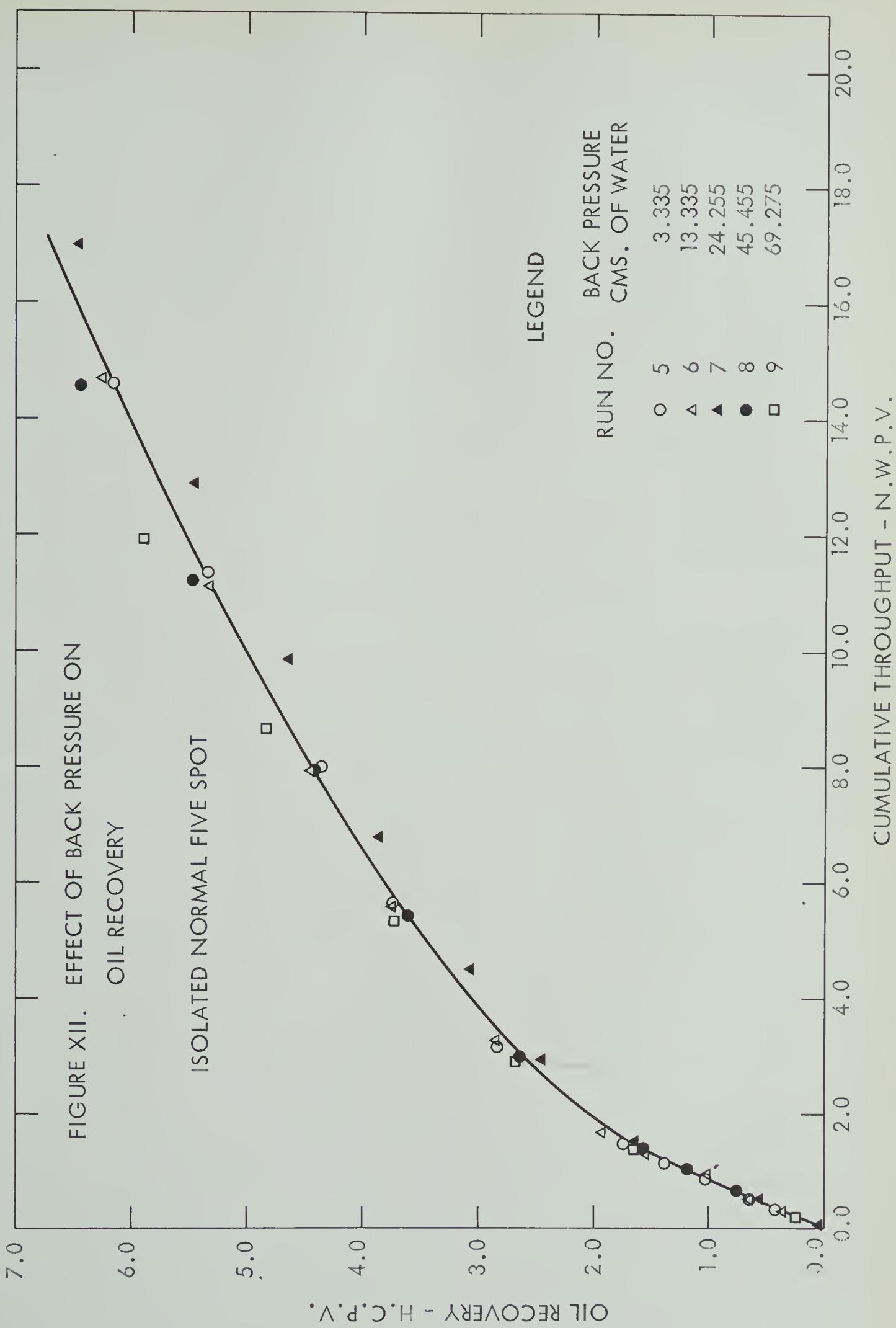
In order to establish that parameter 9 remained constant, a check was made on the reproducibility of experimental results. The experimental runs 8 and 10 were conducted under exactly identical conditions. The results obtained are plotted in Figure XI. It may be observed that, within experimental error , the results of Runs 8 and 10 are reproducible.

The results of experimental runs 5-9 were analysed by plotting oil recovery and areal sweep efficiency against cumulative throughput with back pressure as a parameter. Figure XII presents a plot of cumulative throughput versus oil recovery for five different back pressures. It may be noticed that the scatter in the data points in Figure XII becomes pronounced after 9.0 network pore volumes of water have been injected. This is perhaps due to the influence of the boundary on the growth of the pattern. Thus it may be concluded that within experimental error , oil











recovery is independent of back pressure.

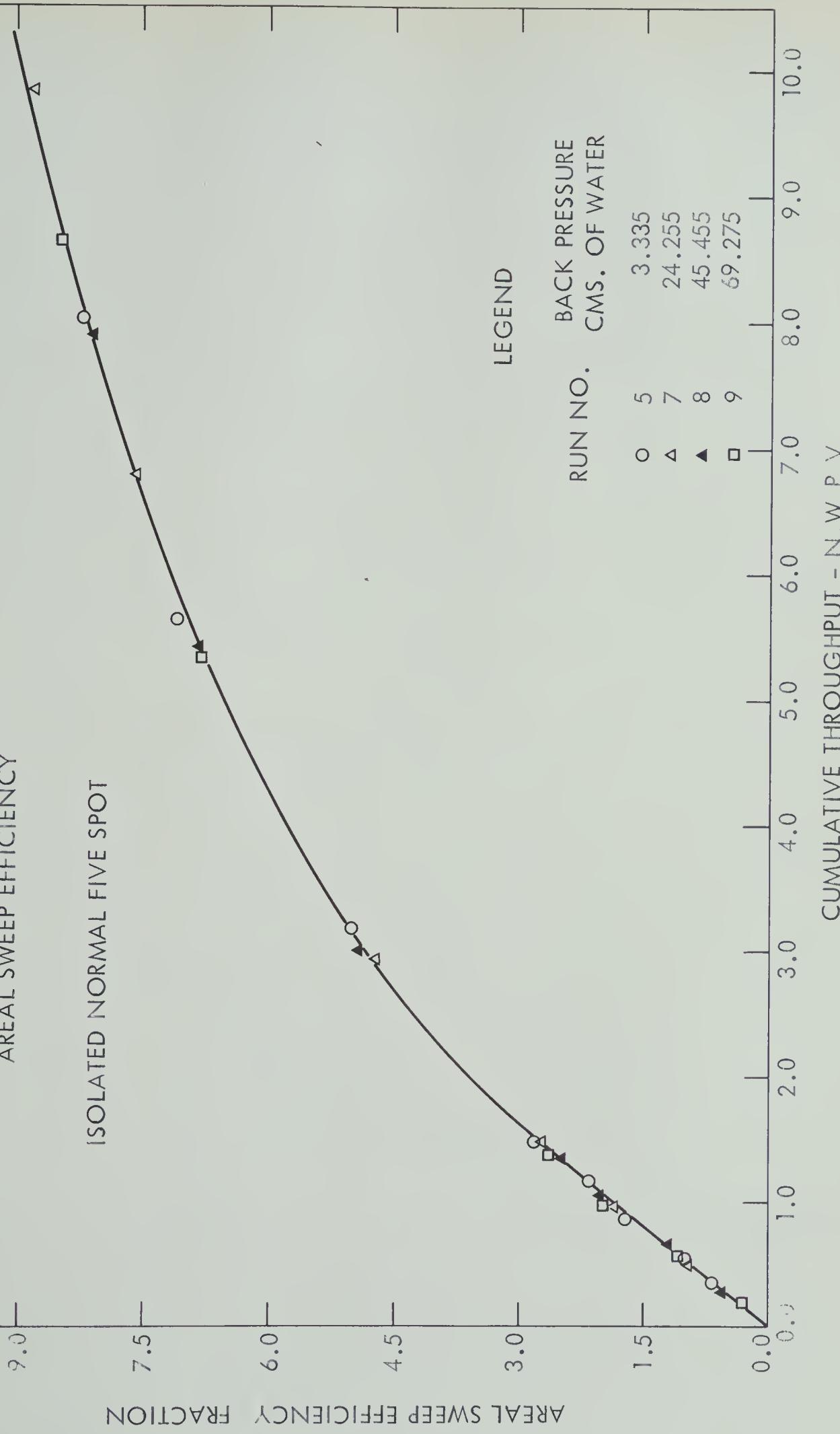
Figure XIII presents a variation of areal sweep efficiency as a function of cumulative throughput at various back pressures. The scatter in this figure is not so pronounced which is due to the fact that data beyond 10.0 network pore volumes of water injected, were not considered. At a cumulative throughput higher than 10.0 network pore volumes there was a significant distortion in the pattern with the result that it was difficult to obtain a meaningful value for the area swept. It may thus be concluded that within experimental error , areal sweep efficiency is not influenced by the pressure.

Figure XIV shows the variation of the instantaneous water-oil ratio as a function of cumulative throughput. The average curve plotted through the data points shows a tendency to stabilize at an instantaneous water oil ratio of 3.5 and a cumulative throughput of 12.4 network pore volumes. This is due to the fact that there is a considerable oversweep in the case of an isolated pattern. This phenomenon of oversweep causes the production of additional oil from freshly contacted areas, thereby resulting in only a slight increase in the water-oil ratio. However, if the experimental runs were continued for a longer period of time, the oversweep would be controlled by the physical dimensions of the model and a

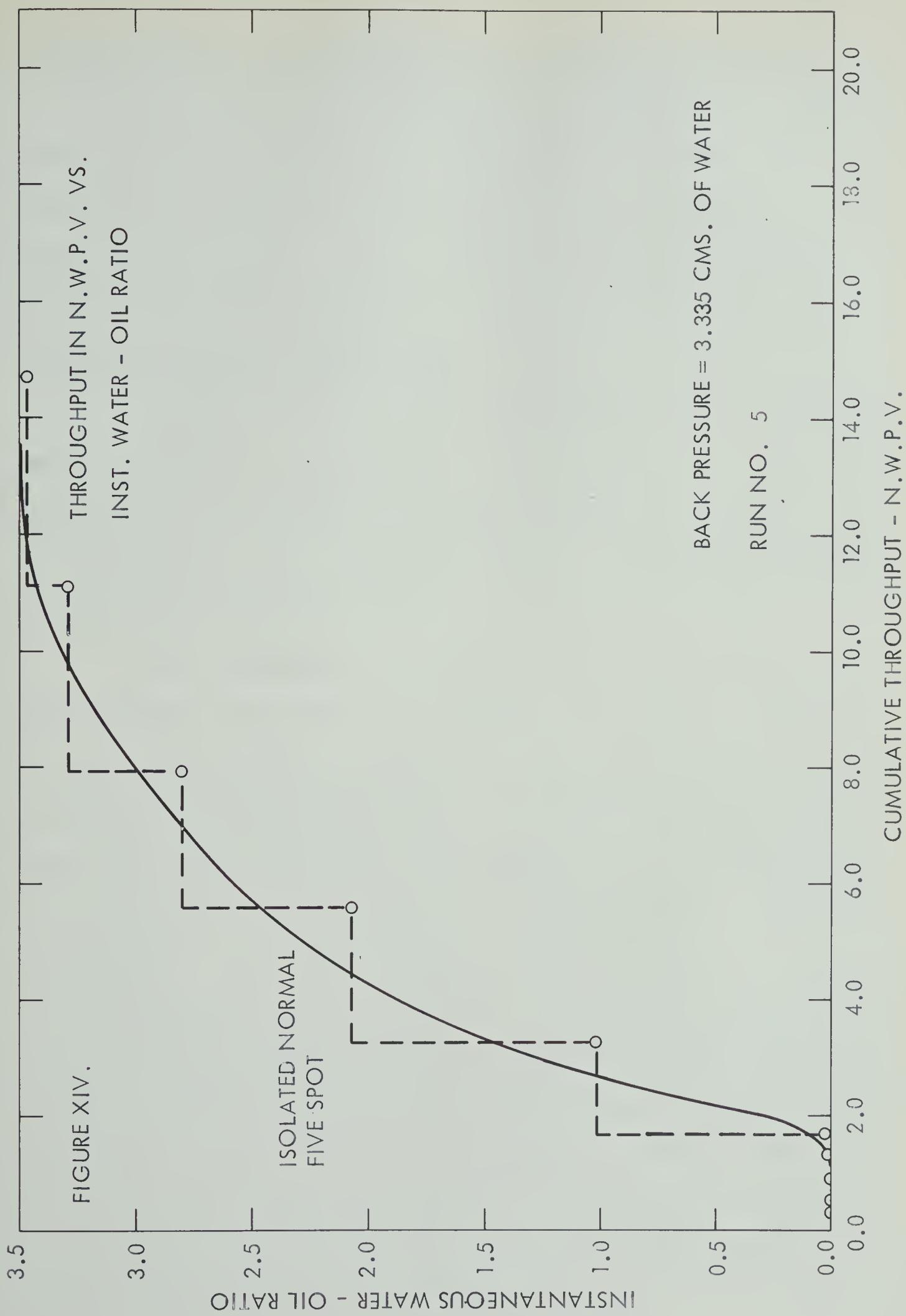


FIGURE XIII. EFFECT OF BACK PRESSURE ON  
AREAL SWEEP EFFICIENCY

ISOLATED NORMAL FIVE SPOT









steady increase in water-oil ratio could be noticed.

An example of the photographs taken during a run showing the area contacted by the injected water, for an isolated five spot and confined five spot patterns is shown in Figures XV - XIX. The white portion in these photographs shows the area contacted by the water.

#### EFFECT OF PATTERN CONFINEMENT

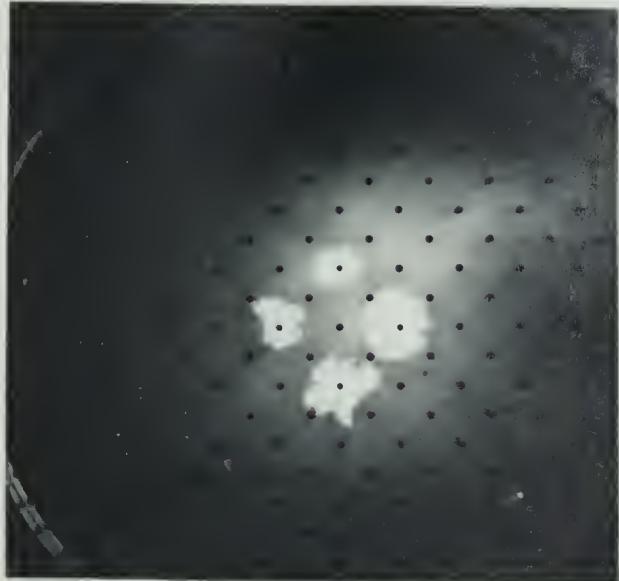
To estimate the extent of oversweep in an isolated pattern, several experimental runs (Runs 11, 12) were conducted on the confined five spot and confined direct line drive patterns. The data obtained are reported in Tables XII and XIII, Appendix C.

The confinement of these patterns was achieved by surrounding the pattern of interest with similar patterns so that equipotential boundaries were established on all sides of the confined pattern. Figures XX and XXI represent the effect of such a confinement on oil recovery and areal sweep efficiency for a five spot pattern. The data required to plot these figures appear in Tables XIV and XV of Appendix C. It may be observed in Figure XX that the oil recovery for the confined five spot pattern stabilizes at a cumulative throughput of 4.6 network pore volumes, whereas it shows an increasing trend in the case of an isolated five spot pattern. This may be explained on the basis that surrounding a pattern with similar

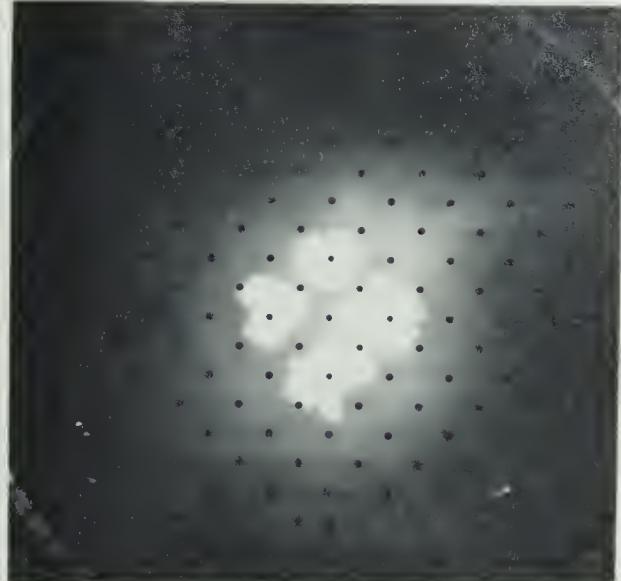


PATTERN GROWTH IN CASE OF ISOLATED NORMAL FIVE SPOT PATTERN  
RUN NO. - 7

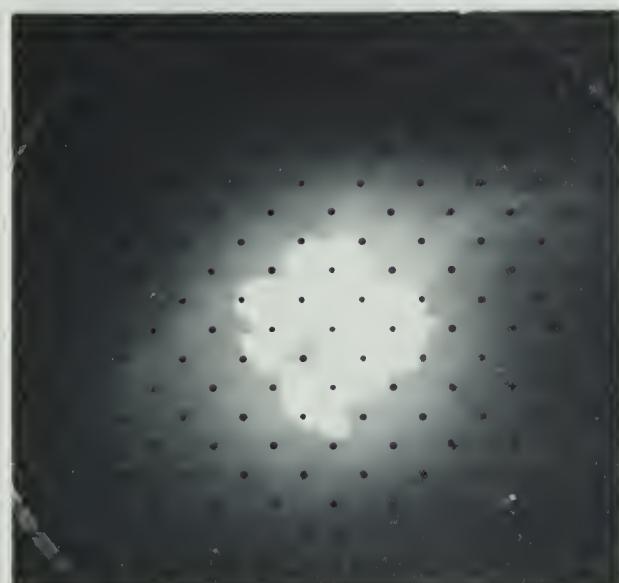
Injection Rate: 345 ccs/hr/well



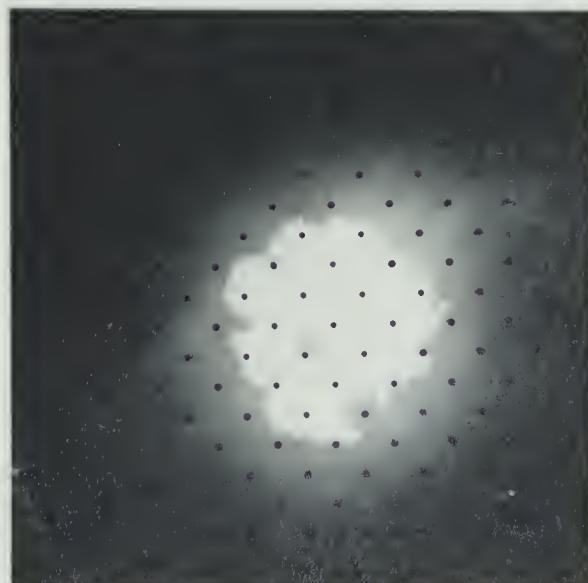
T:1.38 mins



T:2.09 mins



T:4.14 mins



T:6.29 mins

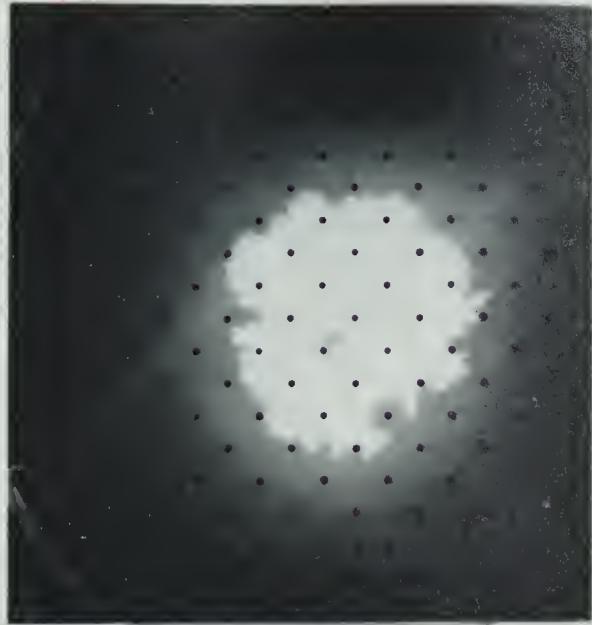
T: refers to the total time elapsed after start

FIGURE XV

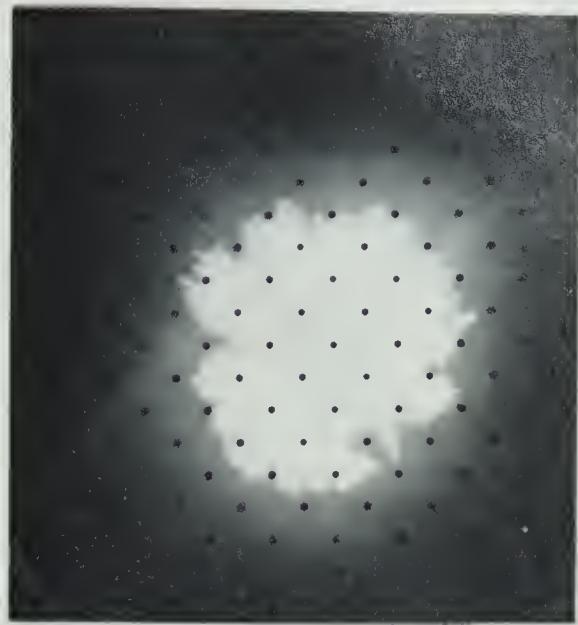


PATTERN GROWTH IN CASE OF ISOLATED NORMAL FIVE SPOT  
RUN NO. - 7

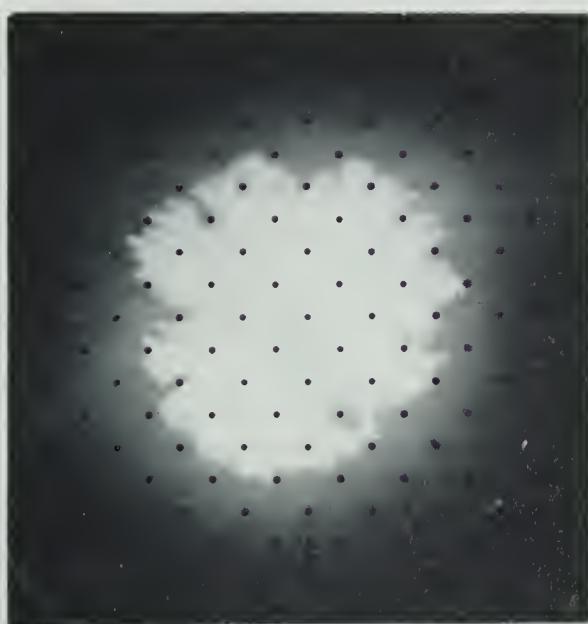
Injection Rate: 345 ccs/hr/well



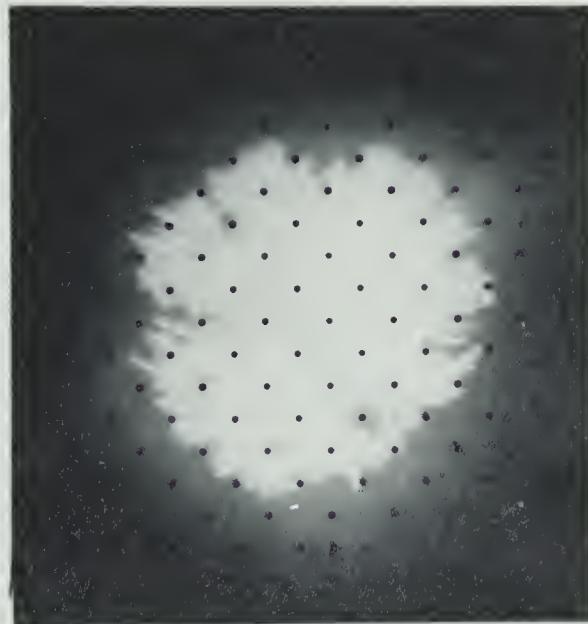
T:9.55 mins



T:13.81 mins



T:18.21 mins



T:23.87 mins

T: refers to the total time elapsed after start

FIGURE XVI



PATTERN GROWTH IN CASE OF CONFINED FIVE SPOT  
RUN NO. - 11

Injection Rate: 345 ccs/hr/well



T:0.26 mins



T:0.60 mins



T:0.97 mins



T:1.42 mins

T: refers to the total time elapsed after start

FIGURE XVII

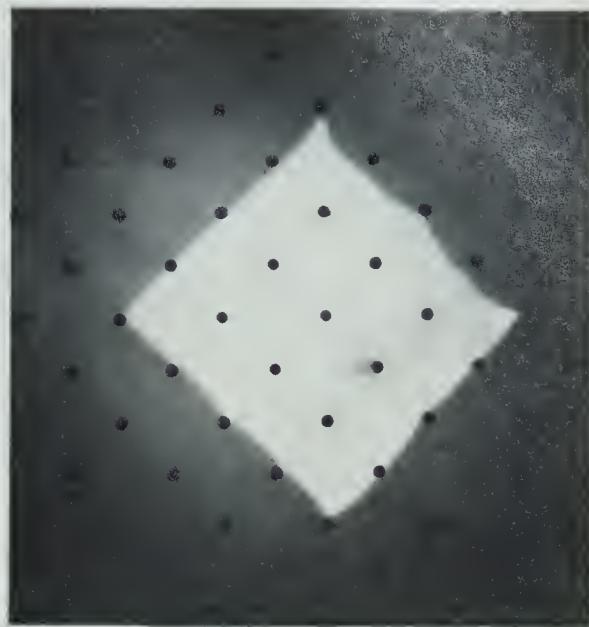


PATTERN GROWTH IN CASE OF CONFINED FIVE SPOT  
RUN NO. - 11

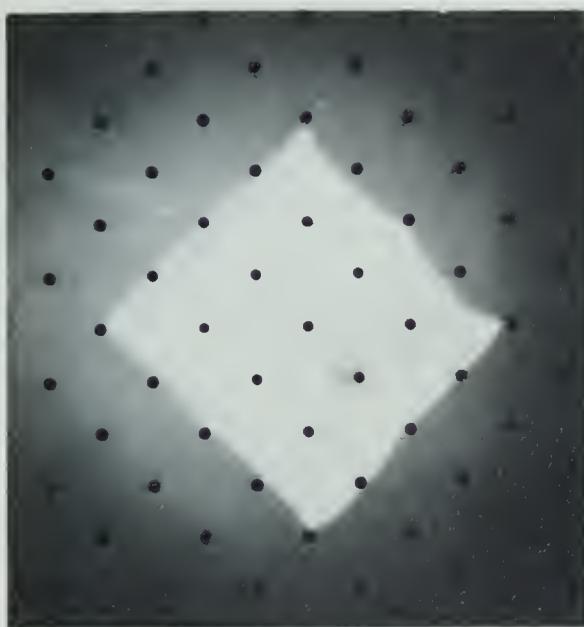
Injection Rate: 345 ccs/hr/well



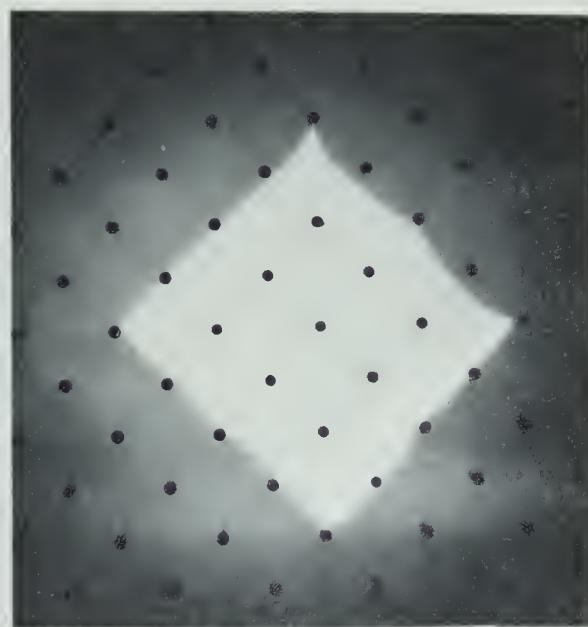
T:1.95 mins



T:2.47 mins



T:3.00 mins



T:3.73 mins

T: refers to the total time elapsed after start

FIGURE XVIII

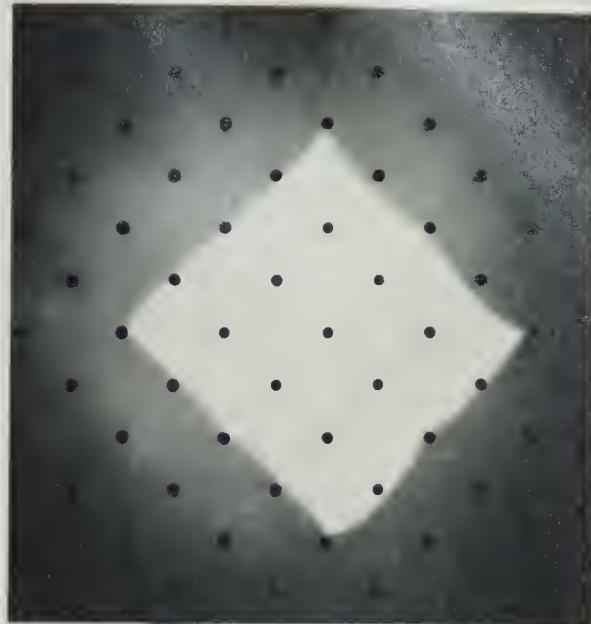


PATTERN GROWTH IN CASE OF CONFINED FIVE SPOT  
RUN NO. - 11

Injection Rate: 345 ccs/hr/well



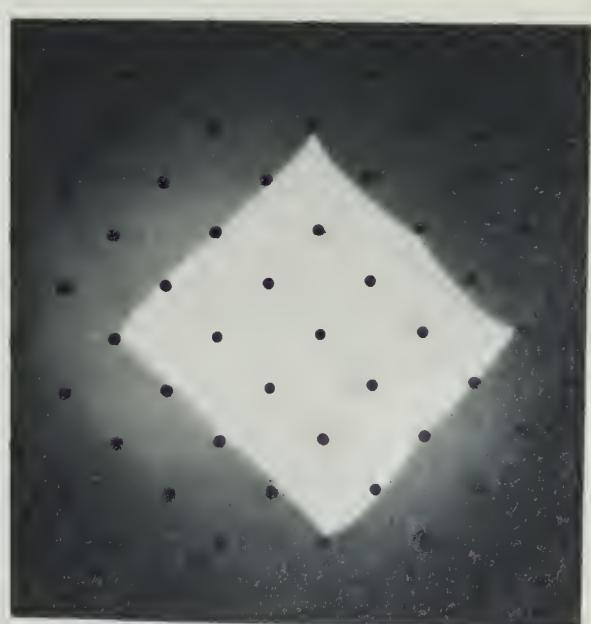
T:7.00 mins



T:4.70 mins



T:5.68 mins

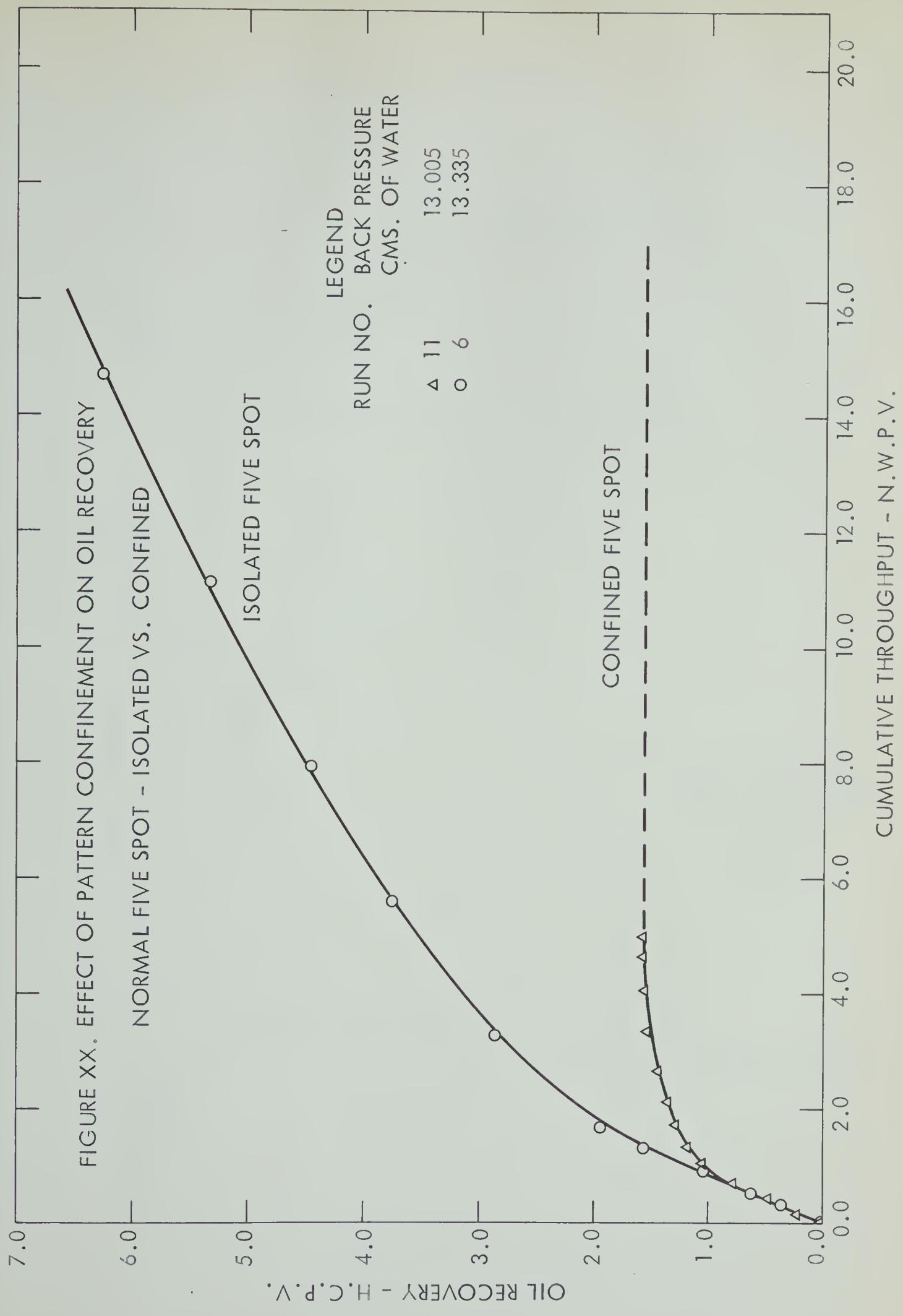


T:6.53 mins

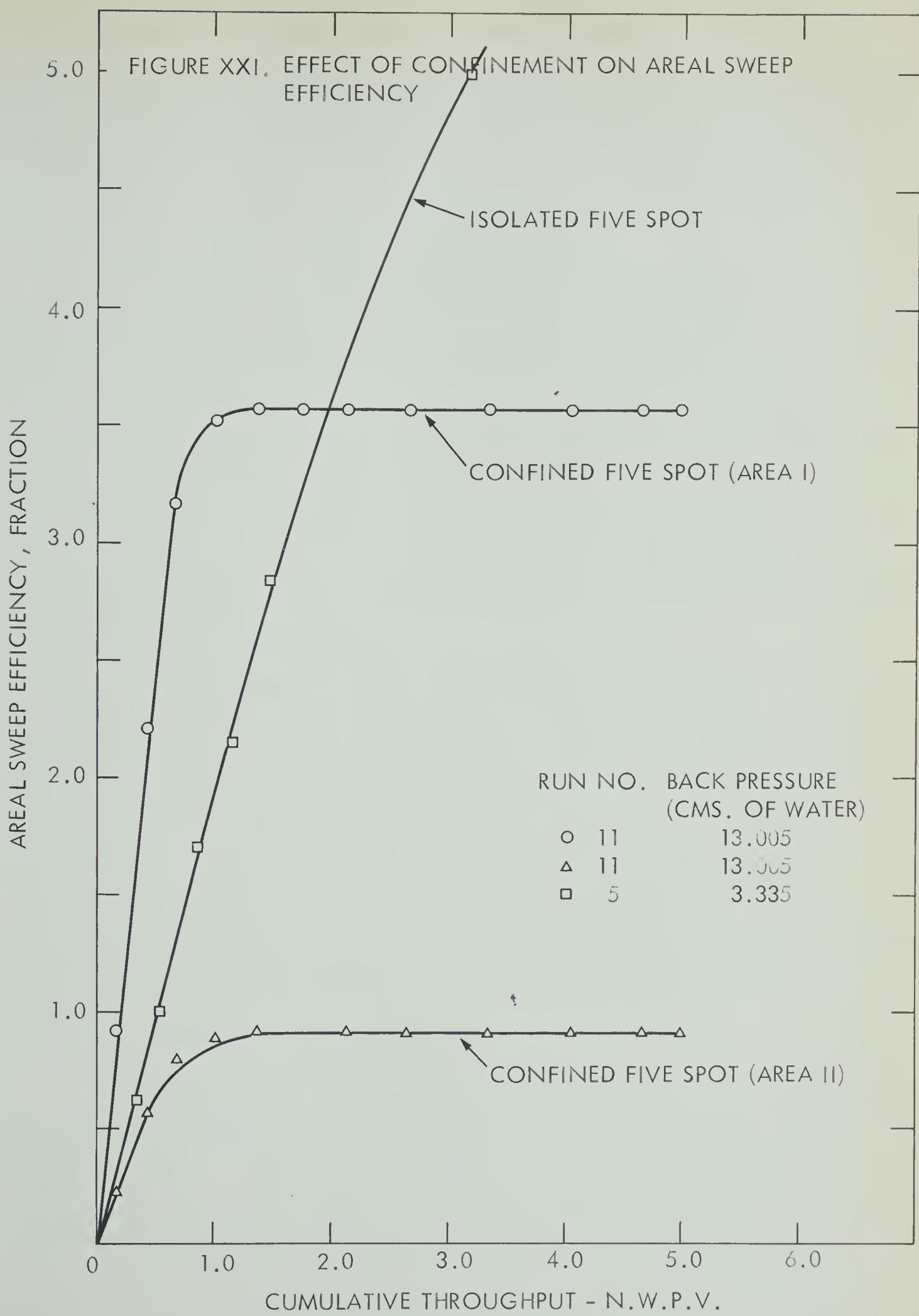
T: refers to the total time elapsed after start

FIGURE XIX











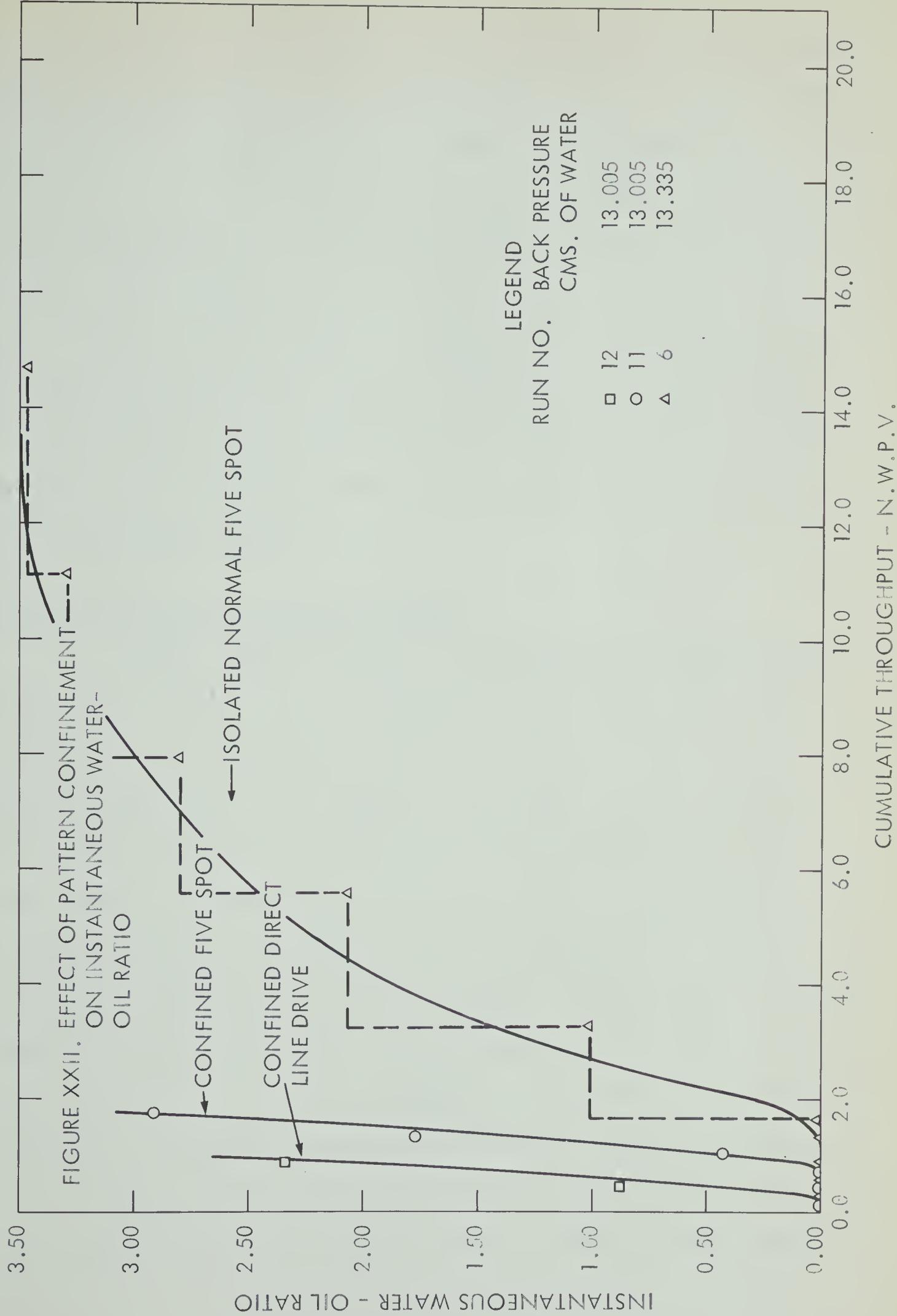
patterns on all sides results in the establishment of equipotential boundaries across which no cross-flow occurs. Thus, the area contacted by the injected water is limited to these boundaries. These boundaries, however, do not exist in the case of an isolated pattern.

A total of 1.55 hydrocarbon pore volumes of oil was recovered from the confined five spot pattern which indicates that in spite of completely surrounding the five spot pattern with similar patterns there was an oversweep. On the basis of the total quantity of oil recovered and irreducible oil saturation at the end of the experimental run, the oversweep was found to be 71.6%. These calculations are reported in Appendix D.

Figure XXI presents areal sweep efficiency as a function of cumulative throughput. The areal sweep efficiency, in this figure, was calculated on the basis of the inside pattern area and the outside pattern area. The inside pattern area (Area I) here implies the actual area of the pattern under study, whereas the outside pattern area (Area II) refers to the area of the white portion visible in the photograph (Figure XIX).

Figure XXII shows the effect of pattern confinement on the relationship between instantaneous water-oil ratio and throughput. It may be noted that water-oil ratio steadily increases as more and more water is injected.





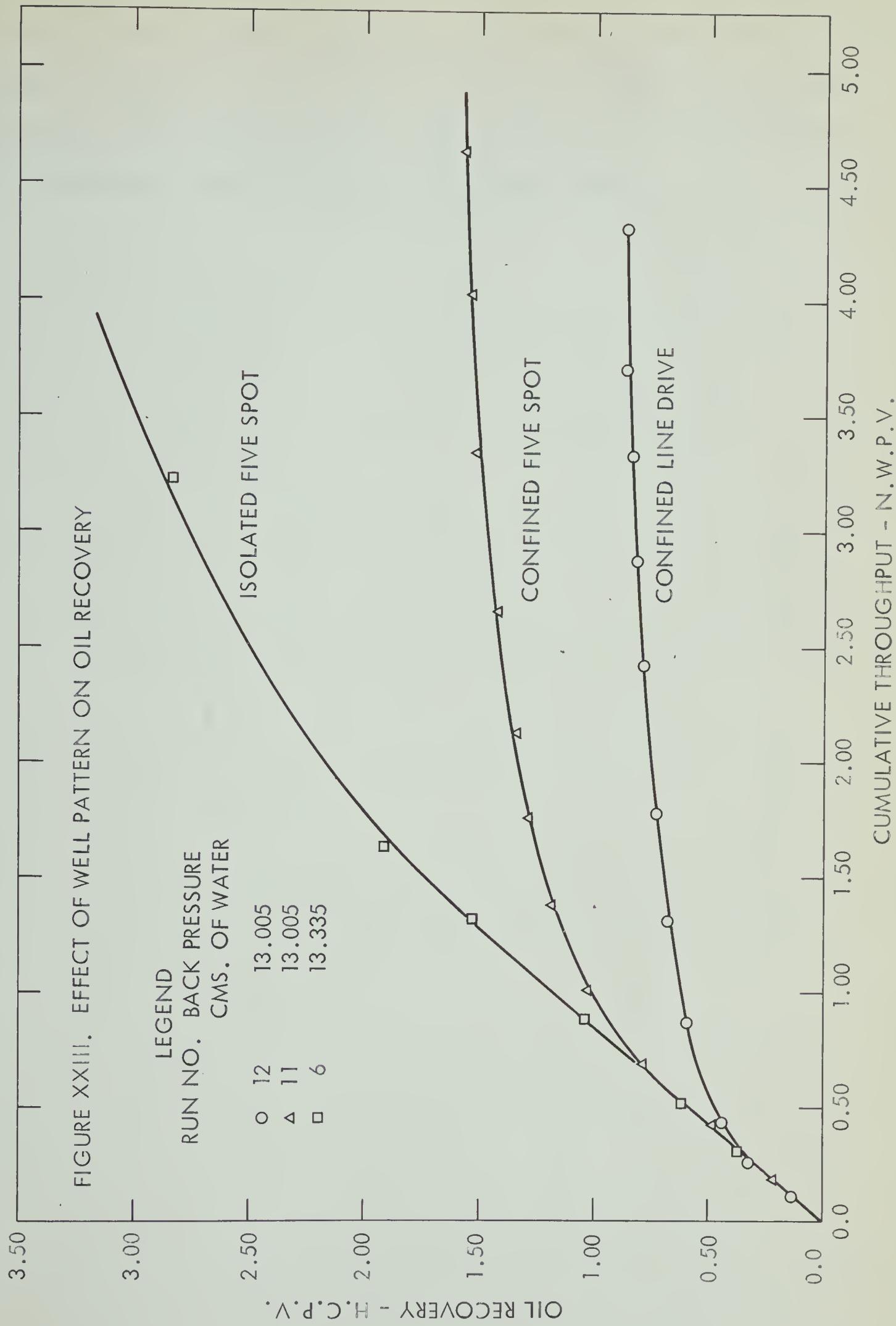


It is expected that after a certain cumulative water injection, the water-oil ratio should rise sharply and no more oil should be produced with further water injection. This behaviour is truly exhibited by the confined five spot and confined direct line drive patterns. However, in the case of the isolated five spot pattern, the increase in water-oil ratio is much slower as compared to the confined case. As explained earlier, this is believed to be due to the oil production from more and more freshly contacted areas which compensated for less and less subordinate production from the swept area.

#### EFFECT OF PATTERN CONFIGURATION

Figure XXIII presents the results of experimental runs 6, 11 and 12 in a concise form and shows the effect of pattern configuration on oil recovery. The obvious conclusion that oil recovery is higher for an isolated five spot pattern as compared to the confined five spot and confined direct line drive patterns, is misleading because there is an oversweep in the case of the isolated five spot and confined five spot patterns. This figure, however, does bring out the fact that as more and more rows of similar patterns are used to surround the pattern of interest, there is a tendency towards achieving complete confinement. In the case of the confined five spot, the central five spot was confined with one row of five spots around its boundary.







and as stated earlier there was still some amount of over-sweep. However, in the case of confined direct line drive, two rows of similar patterns were used and the improvement in confinement may be noticed in Figure XXIII.



### CONCLUSIONS

The results of the present investigation led to the following conclusions.

- 1) There was a definite expansion in the model used by Serra and Bhatia.
- 2) The expansion problem was partially solved in the new model.
- 3) The porous medium used in the present study was reasonably homogeneous.
- 4) Reproducible results were obtained.
- 5) Back pressure was found to have no effect on oil recovery and areal sweep efficiency.
- 6) Surrounding a five spot pattern completely with similar patterns resulted in an oversweep to the extent of 71.6%.
- 7) Complete confinement of a pattern requires more than one row of similar patterns surrounding the pattern under investigation.



RECOMMENDATIONS

- 1) The well design should be altered so that a constant thickness is maintained between the two lucite sheets.
- 2) Use of sintered metal on a well stem, in the packed region, would provide a more uniform flow of fluids through the wells thereby avoiding the cusping (Figure XIX) due to directional orientation of the perforations.
- 3) Use of glass beads and lucite sheets in the present model resulted in non-uniform wettability characteristics of the medium because the beads were water-wet and the interfaces between the glass beads and lucite sheets were oil-wet. This problem can be avoided by using glass sheets, if a water-wet medium is required or lucite beads if it is desired to have an oil-wet medium.
- 4) Use of consolidated porous media should be attempted.



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- (19) Rapoport, L.A., Carpenter, C.W., Jr., and Leas, W.J., "Laboratory Studies of Five Spot Water flood Performance", Trans A.I.M.E. (1958), 213, 113-120.
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## APPENDIX - A

- Average Thickness Correlation
- Calculation of Porosity
- Calculation of Absolute Permeability



AVERAGE THICKNESS CORRELATION

Prior to packing the model with glass beads, a variation in the gap between the confining lucite sheets was noticed. It was observed that the gap was equal to the thickness of the lucite spacer ring at the periphery of the model and decreased towards the center. This variation was attributed to the following reasons.

1. small irregularities in the inside surfaces of the lucite sheets
2. sagging in the model due to the weight of the lucite sheets.

In order to measure this variation, a travelling microscope was used and the gap between the lucite sheets determined by focussing the microscope on the bottom surface of the top plate and then on the top surface of the bottom plate. This measurement was carried out along four diameters of the model and the results averaged to get a representative figure for the variation in thickness. The data obtained were as follows.

<u>RADIUS (From Center)</u>	<u>AVG. THICKNESS</u>
<u>Inches</u>	<u>Cms.</u>
22.6400	0.6158
18.8660	0.5565
15.0930	0.5108
11.3200	0.4830 contd.



RADIUS (From Center)	Avg. Thickness
<u>Inches</u>	<u>Cms.</u>
7.5466	0.4486
3.7730	0.4233
0.0000	0.4222
3.7730	0.4210
7.5466	0.4494
11.3200	0.4600
15.0930	0.5104
18.8660	0.5425
22.6400	0.6133

CALCULATION OF POROSITY1. Measurement of Matrix Volume

Total weight of glass beads added = 9122.39 gms.

Specific gravity of glass beads = 2.515

$$\therefore \text{Matrix Volume of the pack} = \frac{9122.39}{2.515}$$

$$= 3630.42 \text{ c.cs.}$$

Bulk Volume of the model = 5711.0 c.cs.  
(measured by filling the  
model with water)

$$\therefore \text{Porosity, \%} = \frac{\text{Bulk Volume} - \text{Matrix Volume}}{\text{Bulk Volume}} \times 100$$

$$= \frac{5711.0 - 3630.42}{5711.0} \times 100$$

$$= 36.43\%$$



## 2. Measurement of Pore Volume

The pore volume of the model was determined by timing the process of saturating the model completely with water. Knowing the injection rate and the total time, pore volume could be easily calculated.

Time required to completely saturate the medium =  
10.40 hrs.

Injection rate = 200.0 c.cs/hr.

$$\therefore \text{Pore Volume} = 200 \times 10.40 \text{ c.cs.} \\ = 2080.00 \text{ c.cs.}$$

$$\therefore \text{Porosity} = \frac{2080.0}{5711.0} \times 100 \\ = 36.425\%$$

## CALCULATION OF ABSOLUTE PERMEABILITY

The porous medium was completely saturated with water. A five spot pattern was then used to measure absolute permeability. The calculations were performed using Muskat's steady state equation for a five-spot pattern.

### Muskat's Equation:

$$K_w = \frac{Q \mu_w (\ln(d/r_w) - 0.619)}{0.003541 \Delta P h}$$

where,

$K_w$  = effective permeability to water, mds.

$Q$  = rate of injection, bbls/day



$\mu_w$  = viscosity of water, cps.

$d$  = distance between injection well and production well,  
ft.

$r_w$  = well-bore radius, ft.

$\Delta P$  = Pressure differential, psi.

$h$  = formation thickness, ft.

In the present model,

$$\begin{aligned} Q &= 532.0 \text{ c.cs/hr/well} = 2128.0 \text{ c.cs/hr} \\ &= 0.3205 \text{ bbls/day} \end{aligned}$$

$$\begin{aligned} \Delta P &= 5.10 \text{ cms. of mercury} \\ &= 0.987 \text{ psi.} \end{aligned}$$

$$\begin{aligned} h &= \text{Average thickness for } 11.3125 \text{ inch side five spot} \\ &= 0.4517 \text{ cms.} \\ &= 0.01482 \text{ ft.} \end{aligned}$$

$$\mu_w = 0.970 \text{ cps.}$$

$$\begin{aligned} r_w &= 0.0485 \text{ inch} \\ &= 0.00404 \text{ ft.} \end{aligned}$$

$$\begin{aligned} d &= 8 \text{ inches} \\ &= 0.6670 \text{ ft.} \end{aligned}$$

$$d/r_w = 0.6670/0.00404 = 165.20$$

$$\ln(d/r_w) = \ln(165.20) = 5.230$$

$$\begin{aligned} \therefore K_w &= \frac{0.3205 \times 0.970 \times (5.230 - 0.619)}{0.003541 \times 0.01482 \times 0.987} \\ &= 27,700 \text{ mds.} \end{aligned}$$



APPENDIX - B

- Calculation of critical injection rate.



BASIC DATA REQUIRED TO USE THE METHOD

The following data are required to predict the performance of any pattern using the modified scheme:

1. Relative permeability versus saturation curves.
2. Pore volumes and shape factors for each channel.
3. Initial water saturation (connate or irreducible) and irreducible oil saturation.
4. Maximum water-oil ratio at which cut-off is desired.
5. Pressure drop between the injection and the production well for a constant pressure drop case.
6. Injection rate for a constant injection rate case.
7. Time versus injection rate history where both injection rate and pressure drop vary.
8. Absolute permeability of the medium.
9. Oil and water viscosities.



$$\therefore q = \frac{3.5 \times 10^{-3} \times 32.14 \times (27700 \times 0.3643)^{0.5}}{0.970} \text{ bbls/day/ft.}$$

$$= \frac{3.5 \times 10^{-3} \times 32.14 \times 100.45}{0.970} \text{ bbls/day/ft.}$$

$$= 11.649 \text{ bbls/day/ft.}$$

It may be observed that, the critical injection rate, as expressed here, depends upon the thickness of the porous medium. Thus the higher the value used for thickness, the higher is the resulting injection rate. A maximum average thickness of 0.5000 cms. was, therefore, selected for calculation purposes.

$$\begin{aligned} \text{Maximum average thickness} &= 0.5000 \text{ cms.} \\ &= 0.0164 \text{ ft.} \end{aligned}$$

$$\begin{aligned} \text{thus } q &= 11.649 \times 0.0164 \\ &= 0.1910 \text{ bbls/day} \\ &= 1268.17 \text{ c.cs/hr.} \end{aligned}$$

This implies that if the injection rate is kept above 1268.17 c.cs/hr., the resulting floods will be stabilized and will no longer be rate - sensitive. The minimum injection rate used in this study was 1380.0 c.cs/hr.



## APPENDIX - C

- Measurement of Expansion in the old model.
- Measurement of Expansion in the new model.
- Results of Radial flow experiment.
- Experimental data on isolated five spot system at five different back pressures
- Calculation of Oil Recovery and Areal Sweep Efficiency for isolated five spot pattern.
- Calculation of Oil Recovery and Areal Sweep Efficiency for confined five spot pattern.
- Calculation of Oil Recovery for confined direct line drive pattern.



NOMENCLATURE USED IN THE TABLES

N	= Initial oil in place in the model, c.cs.
L	= Length of one side of the well - pattern, cms.
d	= Distance between the injection and the production wells, cms.
$r_w$	= Well bore radius, cms.
$h_{avg}$	= Average thickness of the packing in the well pattern
q	= Injection rate, c.cs/hr.
$\phi$	= Porosity, %.
$S_{o_r}$	= Residual oil saturation
$S_{o_i}$	= Initial oil saturation
$S_{w_c}$	= Connate water saturation
N.W.P.V.	= Net-work pore volume
H.C.P.V.	= Hydrocarbon pore volume
W.O.R.	= Water - oil ratio



TABLE III  
MEASUREMENT OF EXPANSION IN THE OLD MODEL

INJECTION RATE c.cs/hr	AVERAGE INJECTION PRESSURE cms of Hg	EXPANSION - cms.						CONTRACTION - cms.					
		TOP Pointers			BOTTOM Pointers			TOP Pointers			BOTTOM Pointers		
		Left	Middle	Right	Left	Middle	Right	Left	Middle	Right	Left	Middle	Right
320	15.110	--	--	--	--	--	--	--	--	--	--	--	--
400	21.490	--	--	0.010	0.005	--	--	0.015	--	--	--	--	0.030
480	28.140	0.085	0.010	0.005	--	--	--	--	--	--	0.025	0.010	0.025
560	35.240	0.080	0.015	--	--	--	--	--	--	0.075	0.025	0.015	0.020
640	40.810	0.085	0.015	0.015	--	--	--	--	--	--	0.015	0.030	0.020
800	48.040	0.015	0.040	0.035	--	--	--	--	--	--	0.010	0.025	0.030
960	54.900	0.045	0.080	0.035	--	--	--	--	--	--	0.045	0.065	0.045



TABLE IVMEASUREMENT OF EXPANSION IN THE NEW MODEL

<u>Obs. No.</u>	<u>PRESSURE</u>	<u>VOL. OF OIL INJECTED</u>
	<u>cms. of Water</u>	<u>c.cs.</u>
1	18.50	0.856
2	60.70	2.551
3	77.90	3.427
4	99.70	4.270
5	174.93	7.726
6	201.33	9.535

NET INCREASE IN THICKNESS OF THE MODEL

Referring to figure VIII, a pressure of 70 cms. of water causes an increase of 3.05 c.cs. in the bulk volume of the model.

Internal diameter of the model = 45.281 inches

$$\therefore \text{Net increase in thickness} = \frac{3.05}{\pi/4 \times (45.281 \times 2.54)^2}$$

$$= 0.0003 \text{ cms.}$$



TABLE V

RESULTS - RADIAL FLOW EXPERIMENT

<u>BACK PRESSURE</u> <u>Cms. of Water</u>	<u>Obs. No.</u>	<u>FLOW RATE</u> <u>c.cs/hr.</u>	<u>AVG. PRESSURE DROP</u> <u>cms. of Hg.</u>
9.635	1	161.550	0.370
	2	341.100	0.770
	3	436.620	0.975
	4	520.250	1.180
	5	606.880	1.450
21.240	1	162.884	0.350
	2	327.350	0.700
	3	428.373	0.980
	4	520.570	1.230
	5	604.670	1.450
33.740	1	170.310	0.350
	2	334.910	0.716
	3	433.120	0.967
	4	517.850	1.175
	5	607.820	1.450
54.545	1	168.400	0.350
	2	338.930	0.730
	3	434.360	0.950
	4	518.670	1.175
	5	605.500	1.450



RUN NO. 5

N = 1774.60 c.cs.	$S_o_r = 0.0484$
L = 14.40 cms.	$S_o_i = 0.8530$
$d/r_w = 82.60$	$S_w_c = 0.1470$
$h_{avg.} = 0.4250$ cms.	$q = 345$ c.cs/hr/well
$\phi = 36.43\%$	N.W.P.V. = 32.3116 c.cs.
Avg. Pattern area = 0.0086 units	H.C.P.V. = 32.3116 x 0.8530 = 27.562 c.cs.

Back Pressure = 3.335 cms. of water

## PRODUCTION HISTORY

<u>Obs. No.</u>	<u>Water Prod.</u>		<u>Oil Prod.</u>		<u>Total Prod.</u>	
	Inst.	Cum.	Inst.	Cum.	Inst.	Cum.
1	0.0	0.0	11.10	11.10	11.10	11.10
2	0.0	0.0	6.50	17.60	6.50	17.60
3	0.0	0.0	10.40	28.00	10.40	28.00
4	0.0	0.0	10.30	38.30	10.30	38.30
5	0.0	0.0	9.60	47.90	9.60	47.90
6	25.0	25.0	30.00	77.90	55.00	102.90
7	55.0	80.0	24.50	102.40	79.50	182.40
8	59.0	139.0	17.50	119.90	76.50	258.90
9	82.0	221.0	27.00	146.90	109.00	367.90
10	81.0	302.0	23.00	169.90	104.00	471.90
11	114.0	416.0	33.00	202.90	147.00	618.90
12	139.0	555.0	38.00	240.90	177.00	795.90
*	28.0	583.0	7.00	247.90	35.00	830.90

\* Production after injection ceased.



TABLE VIRun No. 5CALCULATION OF OIL RECOVERY AND AREAL SWEEP EFFICIENCY

Obs. No.	Cum. Throughput	Inst. W.O.R.	Oil Recovery	Planimeter Area	Sweep Eff.
				H.C.P.V.	FRACTION
1	0.344	0.000	0.403	0.0053	0.620
2	0.545	0.000	0.639	0.0086	1.000
3	0.867	0.000	1.016	0.0146	1.700
4	1.185	0.000	1.389	0.0185	2.150
5	1.482	0.000	1.737	0.0244	2.840
6	3.185	0.830	2.826	0.0430	5.000
7	5.645	2.245	3.716	0.0607	7.060
8	8.013	3.371	4.351	0.0705	8.200
9	11.390	3.040	5.330	--	--
10	14.605	3.522	6.164	--	--
11	19.154	3.455	7.362	--	--



RUN NO. 6ISOLATED NORMAL FIVE SPOT

N = 1781.40 c.cs.

S<sub>o</sub><sub>r</sub> = 0.0503

L = 14.40 cms.

S<sub>o</sub><sub>i</sub> = 0.8562d/r<sub>w</sub> = 82.60S<sub>w</sub><sub>c</sub> = 0.1483h<sub>avg.</sub> = 0.4250 cms.

q = 345 c.cs/hr/well

ϕ = 36.43%

N.W.P.V. = 32.3116 c.cs.

$$\begin{aligned} \text{H.C.P.V.} &= 32.3116 \times 0.8562 \\ &= 27.6652 \text{ c.cs.} \end{aligned}$$

Back Pressure = 13.335 cms. of waterPRODUCTION HISTORY

<u>Obs. No.</u>	<u>Water Prod.</u>		<u>Oil Prod.</u>		<u>Total Prod.</u>	
	Inst.	Cum.	Inst.	Cum.	Inst.	Cum.
1	0.00	0.00	9.90	9.90	9.90	9.90
2	0.00	0.00	7.00	16.90	7.00	16.90
3	0.00	0.00	11.90	28.80	11.90	28.80
4	0.00	0.00	14.00	42.80	14.00	42.80
5	0.20	0.20	10.00	52.80	10.20	53.00
6	26.00	26.20	25.50	78.30	51.50	104.50
7	52.00	78.20	24.50	102.80	76.50	181.00
8	56.00	134.20	20.00	122.80	76.00	257.00
9	79.00	213.20	24.00	146.80	103.00	360.00
10	90.00	303.20	26.00	172.80	116.00	476.00
11	108.00	411.20	29.00	201.80	137.00	613.00
12	138.00	549.20	37.00	238.80	175.00	788.00
*	37.00	586.20	8.00	246.80	45.00	833.00

\* Production after injection ceased.



TABLE VII

Run No. 6

CALCULATION OF OIL RECOVERY AND AREAL SWEEP EFFICIENCY

Obs. No.	Cum. Throughput	Inst. W.O.R.	Oil Recovery	Planimeter Area	Sweep	
					N.W.P.V.	H.C.P.V.
1		0.306	0.00	0.357	---	---
2		0.523	0.00	0.611	---	---
3		0.891	0.00	1.041	---	---
4		1.325	0.00	1.548	---	---
5		1.640	0.02	1.908	---	---
6		3.234	1.02	2.830	---	---
7		5.600	2.12	3.720	---	---
8		7.954	2.80	4.440	---	---
9		11.140	3.29	5.306	---	---
10		14.732	3.46	6.250	---	---
11		18.972	3.72	7.294	---	---
12		24.388	3.73	8.632	---	---



RUN NO. 7ISOLATED NORMAL FIVE SPOT

$N = 1831.70 \text{ c.cs.}$        $S_{O_r} = 0.0716$   
 $L = 14.40 \text{ cms.}$        $S_{O_i} = 0.8804$   
 $d/r_w = 82.60$        $S_{w_c} = 0.1196$   
 $h_{avg.} = 0.4250 \text{ cms.}$        $q = 345 \text{ c.cs/hr/well}$   
 $\phi = 36.43\%$        $N.W.P.V. = 32.3116 \text{ c.cs.}$   
 Avg. Pattern area = 0.0075      H.C.P.V. =  $32.3116 \times 0.8804$   
 $= 28.450 \text{ c.cs.}$

Back Pressure = 24.255 cms. of water

PRODUCTION HISTORY

Obs. No.	Water Prod.		Oil Prod.		Total Prod.	
	Inst.	Cum.	Inst.	Cum.	Inst.	Cum.
1	0.0	0.0	1.50	1.50	1.50	1.50
2	0.0	0.0	15.00	16.50	15.00	16.50
3	0.0	0.0	15.20	31.70	15.20	31.70
4	0.50	0.50	15.90	47.60	16.40	48.10
5	24.60	25.10	22.50	70.10	47.10	95.20
6	32.00	57.10	17.50	87.60	49.50	144.70
7	52.50	109.60	22.50	110.10	75.00	219.70
8	75.50	185.10	22.50	132.60	98.00	317.70
9	79.00	264.10	22.20	154.80	101.20	418.90
10	102.00	366.10	28.00	182.80	130.00	548.90
11	163.00	529.10	42.00	224.80	205.00	753.90
*	52.00	581.10	12.00	236.80	64.00	817.90

\* Production after injection



TABLE VIII

Run No. 7

## CALCULATION OF OIL RECOVERY AND AREAL SWEEP EFFICIENCY

Obs. No.	Cum. Throughput	Inst. W.O.R.	Oil Recovery	Planimeter Area	Sweep Eff. Fraction	
					H.C.P.V.	
1	0.046	0.000	0.0520	---	---	---
2	0.510	0.000	0.5790	0.0071	0.950	
3	0.981	0.000	1.1140	0.0137	1.820	
4	1.490	0.031	1.6700	0.0206	2.750	
5	2.950	1.093	2.4650	0.0352	4.700	
6	4.480	1.830	3.0800	0.0472	6.300	
7	6.800	2.330	3.8700	0.0566	7.550	
8	9.830	3.360	4.6600	0.0660	8.800	
9	12.960	3.560	5.4400	---	---	
10	16.990	3.640	6.4300	---	---	
11	23.330	3.880	7.9100	---	---	



RUN NO. 8ISOLATED NORMAL FIVE SPOT

$N = 1812.90 \text{ c.cs.}$        $S_{or} = 0.0560$   
 $L = 14.40 \text{ cms.}$        $S_{oi} = 0.8713$   
 $d/r_w = 82.60$        $S_{wc} = 0.1287$   
 $h_{avg.} = 0.4250 \text{ cms.}$        $q = 345 \text{ c.cs/hr/well}$   
 $\phi = 36.43\%$        $N.W.P.V. = 32.3116 \text{ c.cs.}$   
 Avg. Pattern area = 0.0077 uts. H.C.P.V. =  $32.3116 \times 0.8713$   
 $= 28.153 \text{ c.cs.}$

Back Pressure = 45.455 cms. of waterPRODUCTION HISTORY

<u>Obs. No.</u>	<u>Water Prod.</u>		<u>Oil Prod.</u>		<u>Total Prod.</u>	
	Inst.	Cum.	Inst.	Cum.	Inst.	Cum.
1	0.00	0.00	9.60	9.60	9.60	9.60
2	0.00	0.00	11.50	21.10	11.50	21.10
3	0.00	0.00	12.40	33.50	12.40	33.50
4	0.00	0.00	10.50	44.00	10.50	44.00
5	23.00	23.00	30.00	74.00	53.00	97.00
6	51.00	74.00	27.00	101.00	78.00	175.00
7	56.50	130.50	23.50	124.50	80.00	255.00
8	78.00	208.50	30.00	154.50	108.00	363.00
9	82.00	290.50	27.00	181.50	109.00	472.00
10	109.00	399.50	34.00	215.50	143.00	615.00
11	142.00	541.50	39.00	254.50	181.00	796.00
*	38.00	579.50	13.00	267.50	51.00	847.00

\* Production after injection ceased



TABLE IXRun No. 8CALCULATION OF OIL RECOVERY AND AREAL SWEEP EFFICIENCY

<u>Obs. No.</u>	<u>Cum. Throughput</u>	<u>Inst. W.O.R.</u>	<u>Oil Recovery</u>	<u>Planimeter Area</u>	<u>Sweep Eff.</u>	<u>Fraction</u>
	<u>N.W.P.V.</u>		<u>H.C.P.V.</u>			
1	0.297	0.00	0.341	0.0045	0.579	
2	0.653	0.00	0.750	0.0092	1.200	
3	1.037	0.00	1.190	0.0153	1.990	
4	1.362	0.00	1.563	0.0194	2.520	
5	3.000	0.77	2.630	0.0380	4.940	
6	5.420	1.89	3.588	0.0528	6.851	
7	7.900	2.40	4.420	0.0624	8.100	
8	11.230	2.60	5.490	--	--	
9	14.600	3.04	6.450	--	--	
10	19.030	3.20	7.660	--	--	
11	24.640	3.64	9.044	--	--	



RUN NO. 9ISOLATED NORMAL FIVE SPOT

N = 1747.70 c.cs.

So<sub>r</sub> = 0.0392

L = 14.40 cms.

So<sub>i</sub> = 0.8400d/r<sub>w</sub> = 82.60Sw<sub>c</sub> = 0.1600h<sub>avg.</sub> = 0.4250 cms.

q = 345 c.cs/hr/well

ϕ = 36.43%

N.W.P.V. = 32.3116 c.cs.

$$\begin{aligned} \text{Avg. Pattern area} &= 0.0085 \text{ units H.C.P.V.} = 32.3116 \times \\ &\quad 0.8400 \\ &= 27.142 \text{ c.cs.} \end{aligned}$$

Back Pressure = 69.275 cms. of waterPRODUCTION HISTORY

Obs. No.	Water Prod.		Oil Prod.		Total Prod.	
	Inst.	Cum.	Inst.	Cum.	Inst.	Cum.
1	0.00	0.00	6.50	6.50	6.50	6.50
2	0.00	0.00	11.80	18.30	11.80	18.30
3	0.00	0.00	13.50	31.80	13.50	31.80
4	0.00	0.00	13.30	45.10	13.30	45.10
5	21.00	21.00	28.00	73.10	49.00	94.10
6	51.00	72.00	27.50	100.60	78.50	172.60
7	76.00	148.00	30.50	131.10	106.50	279.10
8	78.00	226.00	29.00	160.10	107.00	386.10
9	104.00	330.00	32.00	192.10	136.00	522.10
10	111.00	441.00	35.00	227.10	146.00	668.10
11	136.00	577.00	39.00	266.10	175.00	843.10
*	27.00	604.00	8.00	274.10	35.00	878.10

\* Production after injection ceased



TABLE XRun No. 9CALCULATION OF OIL RECOVERY AND AREAL SWEEP EFFICIENCY

<u>Obs. No.</u>	<u>Cum. Throughput</u>	<u>Inst. W.O.R.</u>	<u>Oil Recovery</u>	<u>Planimeter Area</u>	<u>Sweep Eff. Fraction</u>
	<u>N.W.P.V.</u>		<u>H.C.P.V.</u>		
1	0.2012	0.000	0.2400	0.0254	0.300
2	0.5663	0.000	0.6740	0.0887	1.050
3	0.9842	0.000	1.1720	0.1606	1.990
4	1.3960	0.000	1.6620	0.2239	2.650
5	2.9120	0.750	2.6930	0.3929	4.650
6	5.3420	1.855	3.7060	0.5746	6.800
7	8.6380	2.492	4.8300	0.7140	8.450
8	11.9490	2.690	5.9000	--	--
9	16.1600	3.250	7.0800	--	--
10	20.6700	3.170	8.3700	--	--
11	26.0900	3.487	9.8040	--	--



RUN NO. 10ISOLATED NORMAL FIVE SPOT

N = 1835.80 c.cs.                             $S_{Or} = 0.0678$   
 L = 14.40 cms.                                 $S_{oi} = 0.8828$   
 $d/r_w = 82.60$                                  $S_{wc} = 0.1172$   
 $h_{avg.} = 0.4250$  cms.                         $q = 345$  c.cs/hr/well  
 $\phi = 36.43\%$                                      $N.W.P.V. = 32.3116$  c.cs.  
     $H.C.P.V. = 32.3116 \times$   
    0.8828  
    = 28.50 c.cs.

Back Pressure = 48.100 cms. of water

PRODUCTION HISTORY

Obs. No.	Water Prod.		Oil Prod.		Total Prod.	
	Inst.	Cum.	Inst.	Cum.	Inst.	Cum.
1	0.0	0.0	0.60	0.60	0.60	0.60
2	0.0	0.0	15.80	16.40	15.80	16.40
3	0.0	0.0	15.70	32.10	15.70	32.10
4	0.0	0.0	13.50	45.60	13.50	45.60
5	22.0	22.0	30.00	75.60	52.00	97.60
6	52.0	74.0	26.00	101.60	78.00	175.60
7	57.0	131.0	24.00	125.60	81.00	256.60
8	80.0	211.0	28.00	153.60	108.00	364.60
9	80.0	291.0	23.00	176.60	103.00	467.60
10	103.0	394.0	32.00	208.60	135.00	602.60
11	150.0	544.0	45.00	253.60	195.00	797.60
*	23.0	567.0	8.00	261.60	31.00	828.60

\* Production after injection ceased.



TABLE XIRun No. 10CALCULATION OF OIL RECOVERY

Obs. No.	Cum. Throughput H.W.P.V.	Inst. W.O.R.	Oil Recovery	
			H.C.P.V.	
1	0.0190	0.00	0.0216	0.0216
2	0.5080	0.00	0.5750	0.5750
3	0.9930	0.00	1.1240	1.1240
4	1.4110	0.00	1.6000	1.6000
5	3.0200	0.73	2.655	2.655
6	5.4340	2.00	3.565	3.565
7	7.9410	2.38	4.410	4.410
8	11.2800	2.86	5.390	5.390
9	14.4700	3.48	6.200	6.200
10	18.6500	3.22	7.330	7.330
11	24.6800	3.33	8.900	8.900



RUN NO. 11CONFINED NORMAL FIVE SPOT

N = 1848.40 c.cs.

So<sub>r</sub> = 0.0704

L = 14.40 cms.

So<sub>i</sub> = 0.8884d/r<sub>w</sub> = 82.60Sw<sub>C</sub> = 0.1116h<sub>avg.</sub> = 0.4250 cms.

q = 345 c.cs/hr/well

φ = 36.43%

N.W.P.V. = 32.3116 c.cs.

Avg. Pattern area - I = 0.02115 H.C.P.V. = 32.3116 x

II = 0.0838 units 0.8884

= 28.706 c.cs.

Back Pressure = 13.005 cms. of waterPRODUCTION HISTORY

Obs. No.	Water Prod.		Oil Prod.		Total Prod.	
	Inst.	Cum.	Inst.	Cum.	Inst.	Cum.
1	0.00	0.00	5.90	5.90	5.90	5.90
2	0.00	0.00	7.80	13.70	7.80	13.70
3	0.00	0.00	8.60	22.30	8.60	22.30
4	3.20	3.20	7.10	29.40	10.30	32.60
5	7.80	11.00	4.40	33.80	12.20	44.80
6	9.00	20.00	3.10	36.90	12.10	56.90
7	10.50	30.50	1.50	38.40	12.00	68.90
8	14.70	45.20	2.20	40.60	16.90	85.80
9	19.80	65.00	2.60	43.20	22.40	108.20
10	21.50	86.50	1.00	44.20	22.50	130.70
11	18.60	105.10	0.80	45.00	19.40	150.10
12	11.10	165.10	0.00	45.00	11.00	161.10



TABLE XII  
Run No. 11

CALCULATION OF OIL RECOVERY AND AREAL SWEEP EFFICIENCY

Obs. No.	Cum. Throughput	Inst. W.O.R.	Oil Recovery	Sweep Efficiency, Fraction	
				H.C.P.V.	Area - I Area - II
1	0.1830	0.000	0.2060	0.9267	0.2340
2	0.4240	0.000	0.4770	2.2033	0.5560
3	0.6900	0.000	0.7770	3.1678	0.7995
4	1.0100	0.451	1.0240	3.5083	0.8854
5	1.3860	1.773	1.1820	3.5745	0.9020
6	1.7600	2.903	1.2830	3.5745	0.9020
7	2.1300	7.000	1.3390	3.5745	0.9020
8	2.6600	6.680	1.4180	3.5745	0.9020
9	3.3500	7.620	1.5080	3.5745	0.9020
10	4.0400	21.500	1.5420	3.5745	0.9020
11	4.6500	23.250	1.5650	3.5745	0.9020
12	4.9900	$\infty$	1.5650	3.5745	0.9020



RUN NO. 12CONFINED DIRECT LINE DRIVE

N = 1692.0 c.cs.

So<sub>r</sub> = 0.0596

L = 14.40 cms.

So<sub>i</sub> = 0.8133d/r<sub>w</sub> = 82.60Sw<sub>c</sub> = 0.1867h<sub>avg.</sub> = 0.4250 cms.

q = 345 c.cs/hr/well

∅ = 36.43%

N.W.P.V. = 54.672 c.cs.

$$\begin{aligned} \text{H.C.P.V.} &= 54.672 \times \\ &0.8133 \end{aligned}$$

$$= 44.480 \text{ c.cs.}$$

Back Pressure = 13.005 cms. of waterPRODUCTION HISTORY

<u>Obs. No.</u>	<u>Water Prod.</u>		<u>Oil Prod.</u>		<u>Total Prod.</u>	
	Inst.	Cum.	Inst.	Cum.	Inst.	Cum.
1	0.000	0.000	6.000	6.000	6.00	6.00
2	0.000	0.000	8.000	14.000	8.00	14.00
3	4.932	4.932	5.568	19.568	10.50	24.50
4	16.266	21.198	6.934	26.502	23.20	47.70
5	20.667	41.865	3.433	29.935	24.10	71.80
6	23.131	64.996	2.869	32.804	26.00	97.80
7	32.792	97.788	2.408	35.212	35.20	133.00
8	23.626	121.414	1.274	36.486	24.90	157.90
9	24.099	145.513	1.001	37.487	25.10	183.00
10	19.359	164.872	0.541	38.028	19.90	202.90
11	32.878	197.750	0.622	38.650	33.50	236.40
12	42.450	240.200	0.550	39.200	43.00	279.40



TABLE XIII

Run No. 12

## CALCULATION OF OIL RECOVERY

Obs. No.	Cum. Throughput	N.W.P.V.	Inst. W.O.R.			Oil Recovery	H.C.P.V.
1	0.110		0.000			0.135	
2	0.256		0.000			0.315	
3	0.448		0.886			0.440	
4	0.872		2.346			0.596	
5	1.313		6.030			0.674	
6	1.788		8.060			0.738	
7	2.432		13.618			0.792	
8	2.888		18.545			0.820	
9	3.347		24.076			0.843	
10	3.711		35.780			0.856	
11	4.324		52.858			0.869	
12	5.110		77.200			0.882	



TABLE XIVRuns 6 and 11EFFECT OF PATTERN CONFINEMENT ON OIL RECOVERY

Obs. No.	ISOLATED FIVE SPOT		CONFINED FIVE SPOT	
	N.W.P.V.	H.C.P.V.	Cum. Throughput	N.W.P.V.
1	0.306	0.357		0.1830
2	0.523	0.611		0.4240
3	0.891	1.041		0.6900
4	1.325	1.548		1.0100
5	1.640	1.908		1.3860
6	3.234	2.830		1.7600
7	5.600	3.720		2.1300
8	7.954	4.440		2.6600
9	11.140	9.306		3.3500
10	14.732	6.250		4.0400
11	18.972	7.294		4.6500
12	24.388	8.632		4.9900
			H.C.P.V.	1.5650



## APPENDIX - D

- Calculation of displacement efficiency for:
  - a. Isolated five spot
  - b. Confined five spot
- Calculation of oversweep in confined five spot.
- Calculation of oil recovery from areal sweep efficiency versus cumulative throughput curve.



CALCULATION OF DISPLACEMENT EFFICIENCY FOR ISOLATED NORMALFIVE SPOT

Displacement Efficiency,

$$E_d = \frac{\text{Oil Recovered at any stage}}{\text{Area contacted by flooding water at this stage} \times \text{Average thickness} \times \text{Porosity} \times \text{Initial Oil Saturation}}$$

In the case of Run No. 9,

Porosity = 0.3643

Initial Oil saturation = 0.8400

Referring to Table X it is observed that at a total injection of 5.3420 network pore volumes of water,

Recovery of oil = 3.7060 H.C.P.V.

$$= 3.7060 \times 27.142$$

$$= 100.60 \text{ c.cs.}$$

Areal sweep efficiency = 6.8

$$\therefore \text{Area contacted by flooding water} = 6.8 \times (14.4)^2$$

$$= 1410.10 \text{ sq. cms.}$$

Average thickness for this area, from Appendix A

$$= 0.4533 \text{ cms.}$$

$$\therefore E_d = \frac{100.60}{1410.10 \times 0.4533 \times 0.3643 \times 0.8400} \times 100$$

$$= \underline{51.50\%}$$



CALCULATION OF DISPLACEMENT EFFICIENCY FOR CONFINED FIVE SPOT

In the case of confined five spot the areal sweep corresponding to the oil recovered is undefined because of the manner in which pattern confinement was achieved. As such the following definition of displacement efficiency has been used.

Displacement Efficiency,

$$\begin{aligned} E_d &= \frac{\text{Total oil recovered}}{\text{Oil in place}} \\ &= \frac{(1 - S_{w_c} - S_{o_r})}{(1 - S_{w_i})} \end{aligned}$$

In the case of Run No. 11

$$S_{o_r} = 0.0704$$

$$S_{w_i} = 0.1116$$

$$\begin{aligned} \therefore E_d &= \frac{(1 - 0.1116 - 0.0704)}{(1 - 0.1116)} \times 100 \\ &= \frac{0.8180}{0.8884} \times 100 \\ &= \underline{92.1\%} \end{aligned}$$

It may be noted that this value of displacement efficiency is applicable only when complete flooding of the pattern has occurred.



CALCULATION OF OVERSWEET IN CONFINED FIVE SPOT

In the case of Run No. 11,

$$\text{Porosity} = 0.3643$$

$$\text{Residual oil saturation} = 0.0704$$

$$\text{Connate water saturation} = 0.1116$$

$$\text{H.C.P.V.} = 28.706$$

$$\text{Avg. thickness} = 0.4250 \text{ cms.}$$

Referring to Table XII it is observed that oil recovery stabilizes at 1.5650 H.C.P.V.

Thus total oil recovered from the pattern

$$= 1.5650 \times 28.706$$

$$= 45.0 \text{ ccs.}$$

Oil recovered can also be determined from the following equation:

$$\begin{aligned} \text{Oil recovered} &= \text{Area swept} \times \text{thickness} \times \text{porosity} \times \\ &\quad (1 - \text{Residual oil saturation} - \\ &\quad \text{connate water saturation}) \end{aligned}$$

$$\begin{aligned} \text{Area swept} &= \frac{45.0}{0.4250 \times 0.3643 \times (1 - 0.0704 - 0.1116)} \\ &= 355.31 \text{ sq. cms.} \end{aligned}$$

$$\text{Pattern Area} = 14.4 \times 14.4$$

$$= 207.0 \text{ sq. cms.}$$

$$\begin{aligned} \therefore \text{Areal sweep efficiency} &= \frac{355.31}{207.0} \times 100 \\ &= 171.6\% \end{aligned}$$

$$\text{Thus oversweep} = 171.6 - 100$$

$$= 71.6\%$$

The extra area swept on the basis of this areal sweep efficiency is indicated by the shaded portion on Figure XXIV.



CONFINED FIVE SPOT  
REPRESENTATION OF EXTRA AREA SWEEP

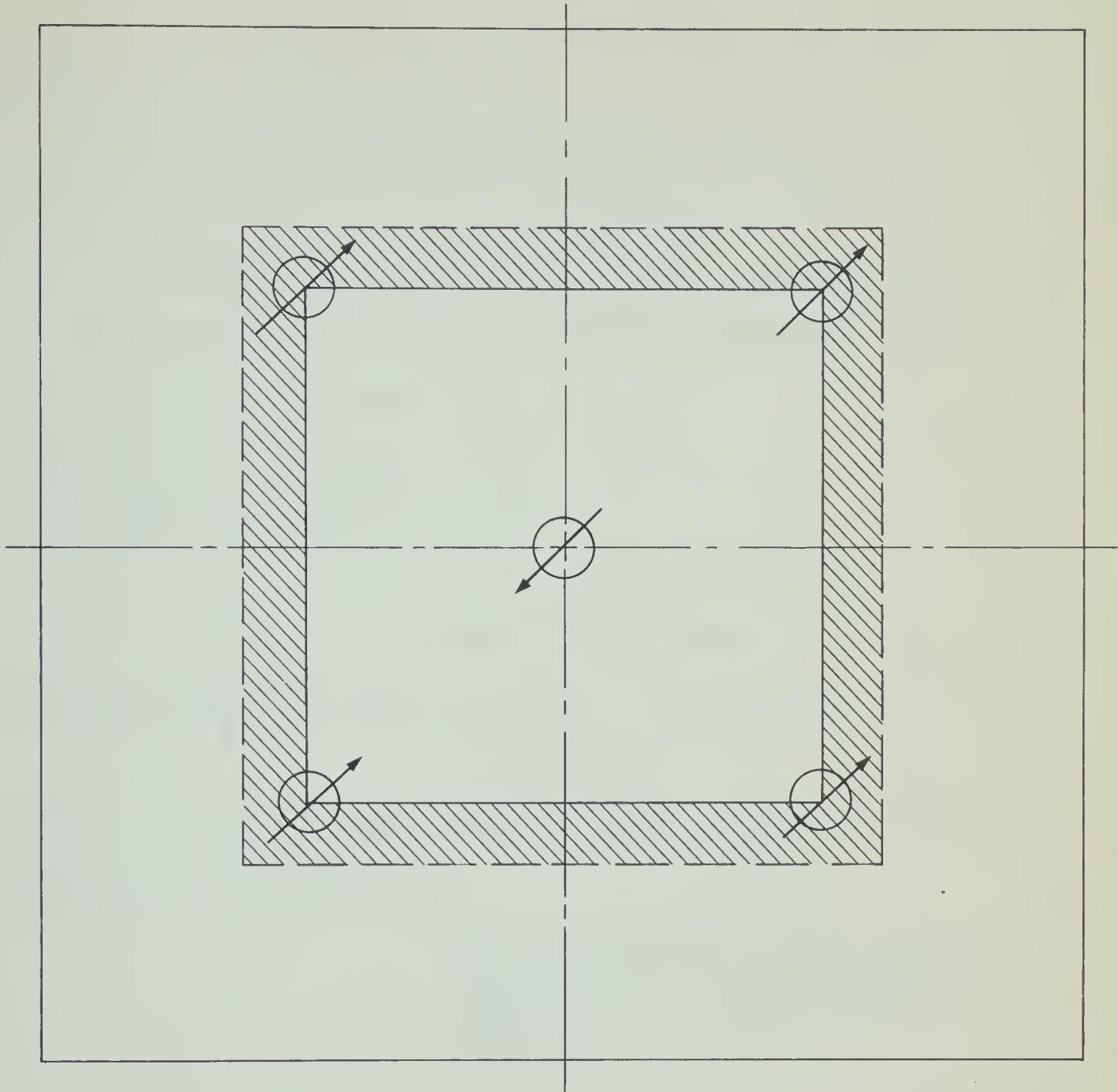


FIGURE - XXIV



CALCULATION OF OIL RECOVERY FROM AREAL SWEEP EFFICIENCY  
VERSUS CUMULATIVE THROUGHPUT CURVE

In the case of Run No. 7 on Isolated Normal Five Spot,

Porosity = 0.3643

Irreducible oil saturation = 0.0716

H.C.P.V. = 28.45 c.cs.

Referring to Table VIII, it is observed that at a total injection of 2.95 network pore volumes of water,

Oil recovered = 2.4650 H.C.P.V.

Areal sweep efficiency = 4.70

Thus, area contacted by flooding

$$\begin{aligned} \text{water} &= 4.70 \times (14.4)^2 \\ &= 974.59 \text{ sq. cms.} \end{aligned}$$

Average thickness for this area, from Appendix A

$$= 0.4400 \text{ cms.}$$

Amount of oil displaced when 974.59 sq. cms. of area is contacted by flooding water

$$= \text{Area contacted} \times \text{avg. thickness} \times \text{porosity} \times (1 - \text{residual oil saturation}) \times \text{displacement efficiency.}$$

$$\begin{aligned} \therefore \text{Oil displaced} &= 974.59 \times 0.440 \times 0.3643 \times (1 - 0.0716) \\ &\quad \times 0.5150 \\ &= \underline{\underline{74.6 \text{ c.cs.}}} \end{aligned}$$



Comparing it to the oil recovered:

Oil recovered = 2.4650 H.C.P.V.

$$= 2.4650 \times 28.45$$

$$= \underline{70.2 \text{ c.cs.}}$$

Thus, it may be observed that, within the experimental errors, the agreement between oil recovered and oil displaced is reasonably close.



P A R T      T W O

SOME MODIFICATIONS TO  
THE HIGGINS - LEIGHTON METHOD



PART TWO

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### INTRODUCTION

The objective of this investigation was to check the compatibility of laboratory results against the results obtained from theoretical prediction, using the properties of the model. Various waterflood prediction methods which have been proposed in the literature from time to time were reviewed. It was decided to use the method suggested by Higgins and Leighton (1) due to its simplicity.

An examination of the Higgins-Leighton (2) method revealed that it is applicable only to the case where a constant pressure drop is maintained between the injection and the production well. Since the experimental work was conducted at constant injection rate conditions, a few modifications had to be made in the original scheme before it could be applied to the present work.



## LITERATURE SURVEY

Waterflood prediction methods give estimates of the following:

- ultimate production to be attained
- time rate of oil production both before and after breakthrough
- time rate of water production after breakthrough

Depending on the purpose of the waterflood prediction, this estimate of performance may range from a thorough analysis of all factors known to affect the flood to a simple rule of thumb for preliminary evaluation. The first step in the development of a prediction method is to overcome the problems caused by significant reservoir heterogeneities. It was recognized very early in the literature that the success or failure of a flood depended upon the permeability stratifications existing within the reservoir. Later on it was discovered that the injection well and its injectivity characteristics were fundamental to the study of such a project. The classical methods which developed from these considerations are those proposed by Dykstra-Parsons (3), Stiles (4), Yuster - Suder - Calhoun (5) and much later by Pratts, Mathews, Jewett, Baker (6).

While these methods were primarily concerned with permeability and injectivity variations, there



developed at the same time an appreciation of the displacement mechanism which was responsible for the overall floodability of a reservoir. The two areas of primary concern were:

- a definition of the macroscopic variables controlling the areal sweep efficiency during and after water breakthrough into the producing well and
- the definition of microscopic variables controlling the displacement of oil by water.

Both these effects are influenced by the oil viscosity and relative permeability characteristics of the porous medium. The former problem, dealing with the definition of macroscopic variables, was more emphasized in the case of pattern floods where the proportion of reservoir volume swept to breakthrough was a variable. Significant contributions along this line evolved in the prediction methods of Muskat (7), Hurst (8), and a little later Slobod-Caudle (9), Dyes - Caudle - Erickson (10), and Caudle-Witte (11).

The fundamental theory developed by Buckley - Leverett (12) for linear flood behavior has been helpful in analyzing physical problems associated with the microscopic displacement mechanism. This theory, however, was not put into a convenient form for pattern flooding until incorporated in the prediction method of Craig,



Geffen and Morse (13).

Since then, the method proposed by Higgins and Leighton (14) has also used this approach with a practical result. This method includes a microscopic formulation of the Buckley-Leverett model and is able to predict the behavior of a flood developed with almost any type of injection/production well arrangement.

Each published prediction method has been more or less slanted towards a definition of one or more primary variables known to affect the water flood performance. Pertinent variables which, however, must be considered in any comprehensive prediction method are:

- Permeability distribution
- Injectivity or Injection rate
- Areal Sweep Efficiency
- Mobility Ratio
- Displacement Mechanism
- Non-uniformities

#### PERMEABILITY DISTRIBUTION

Early prediction methods were concerned with permeability changes due to reservoir stratification. The availability of computers today has virtually eliminated this problem as predictions can now be made rapidly on single layers and superimposed to get the combined stratified reservoir performance. Higgins and Leighton (15)



extended this approach by subdividing a single homogeneous bed into individual flow channels or ducts. Performance calculations are made for each individual channel, summed to get the layer performance and superimposed with the performance from other layers to get the overall reservoir performance.

#### INJECTIVITY OR INJECTION RATE

Time can be introduced into the performance prediction only through a knowledge of the expected injection rate as production continues. Since roughly 80% of the total pressure drop in a five spot pattern occurs in about 4% of the volume near the injection well and 4% near the producer, injection rates calculated from differential pressure are sensitive to viscous effects in the oil and water banks. An excellent analysis of this problem has been presented by Prats et.al. (16). Perhaps the largest single source of error in predicting reservoir performance is the need for complete accountability of injected water. Fractures, casing and packer leaks, improperly abandoned wells, aquifer injection, directional permeability, and regional pressure gradients pose possible reasons for differences between the actual and predicted injectivity and production performance.



### AREAL SWEEP EFFICIENCY

Considerable ingenuity was required to solve the areal sweep problem. Prediction methods involving multi-layered reservoirs have generally resorted to vertical conformance or experience factors to account for volumetric sweepout by the displacing fluid. Single layer prediction methods have relied more on model studies for determining areal sweep efficiency and its dependence on mobility ratio. Earlier prediction methods were more concerned with areal sweep at breakthrough than the variation in sweep out as a function of water cut following breakthrough. It was left to Dyes et.al. (17) to incorporate this factor into the water flood prediction scheme.

### MOBILITY RATIO

The ratio of water mobility to that of oil is a primary variable in predicting water flood performance. Basically it provides a measure of the relative ease with which the injected water can displace oil. Prediction methods use mobility ratio as a correlative variable determined from laboratory studies. A related variable - conductance, defined as the ratio of injection rate  $q$  to pressure drop  $\Delta P$  - accounts for the overall fluid conductivity of the reservoir and has been utilised by Caudle and Witte (18).



## DISPLACEMENT MECHANISM

The final primary variable to influence water-flood performance is the actual displacement mechanism itself which operates to force the oil out of the reservoir. Studies of this variable in the laboratory indicate that for values of injection rate lower than the critical\*, water flood is rate sensitive and, therefore, the amount of oil recovered varies with the injection rate. However, for injection rates higher than the critical; the performance of a water flood is no longer influenced by the rate. Most all prediction methods which account for the displacement mechanism in an acceptable and comprehensive manner use some form of Buckley-Leverett (20) frontal advance theory.

## NON-UNIFORMITIES

It has been said that the reservoir non-uniformity effects are the predominant factors in controlling the overall water flood performance. Therefore, thoroughly evaluated reservoir non-uniformities make up the most important data in any prediction method. Two basic methods for characterizing permeability variations in a reservoir are available.

---

\* critical injection rate is the rate obtained from the scaling coefficient proposed by Rapoport, Carpenter and Leas (19).



- the frequency distribution method, and
- the positional approach.

The frequency distribution method involves arranging the core analysis permeabilities in order from largest to the smallest value. These are ordered regardless of location of the core samples in the well. Then permeabilities are divided into ten or more groups and the permeabilities within any group are averaged to get the average layer permeability. This approach is described by Stiles (21).

The positional approach on the other hand, was first described by Miller and Lents (22) and later by Elkins (23). This requires core analyses from at least two wells. The permeabilities of the core samples in all wells one foot below the top of the pay are averaged and similarly two feet below the top, etc. As a result of this technique the high and low permeabilities are averaged together, yielding a much less extreme permeability profile than that obtained from the frequency distribution method.

Summarising the goals of various prediction methods that have so far been proposed, it can be stated that the best prediction method is one which includes:

- fluid flow effects
- pattern effects
- heterogeneity effects.



Included in the fluid flow effects are saturation gradient which can occur behind the flood front, consideration of the effects of an initial gas saturation and varying fluid conductivity.

Well pattern effects involve consideration of areal sweep efficiency at breakthrough as a function of mobility ratio and the subsequent increase in areal sweep with continued water injection.

Heterogeneity effects include consideration of the non-uniform nature of oil reservoirs, permeability stratification etc. Aside from these three factors, in the case of communicating layers, the best prediction method should also consider the effects of cross flow due to viscous, capillary and/or gravitational forces. Above all it should agree with the actual field performance.



THE HIGGINS-LEIGHTON METHOD

Recently, Higgins and Leighton (24) published a numerical technique for the calculation of fluid displacement for any irregularly bounded porous medium. The method employs the Buckley-Leverett (25) and Welge (26) concepts and is based on the following assumptions:

1. The porous medium considered is homogeneous and has uniform permeability and porosity.
2. A constant known pressure drop is maintained between the injection and the production well.
3. Gravitational and capillarity effects are negligible.
4. The mobility ratio of the system is unity.
5. There is no cross flow between the various channels.
6. Gas saturation is identical in both oil and water banks.
7. Fluids are incompressible.

The approach used is best illustrated by reference to Figure I, which shows the isopotential and streamlines for the five spot flooding network. Here, a potentiometric model has provided detailed information on the positions of the streamlines and isopotentials. It may be noticed that streamlines (1) and (2) enclose a flow channel, and that such a channel could be divided into a number of cells of equal volume between the injection and



FIVE-SPOT FLOODING NETWORK  
SHOWING STREAMLINES AND  
ISOPOTENTIAL LINES FOR UNIT  
MOBILITY RATIO

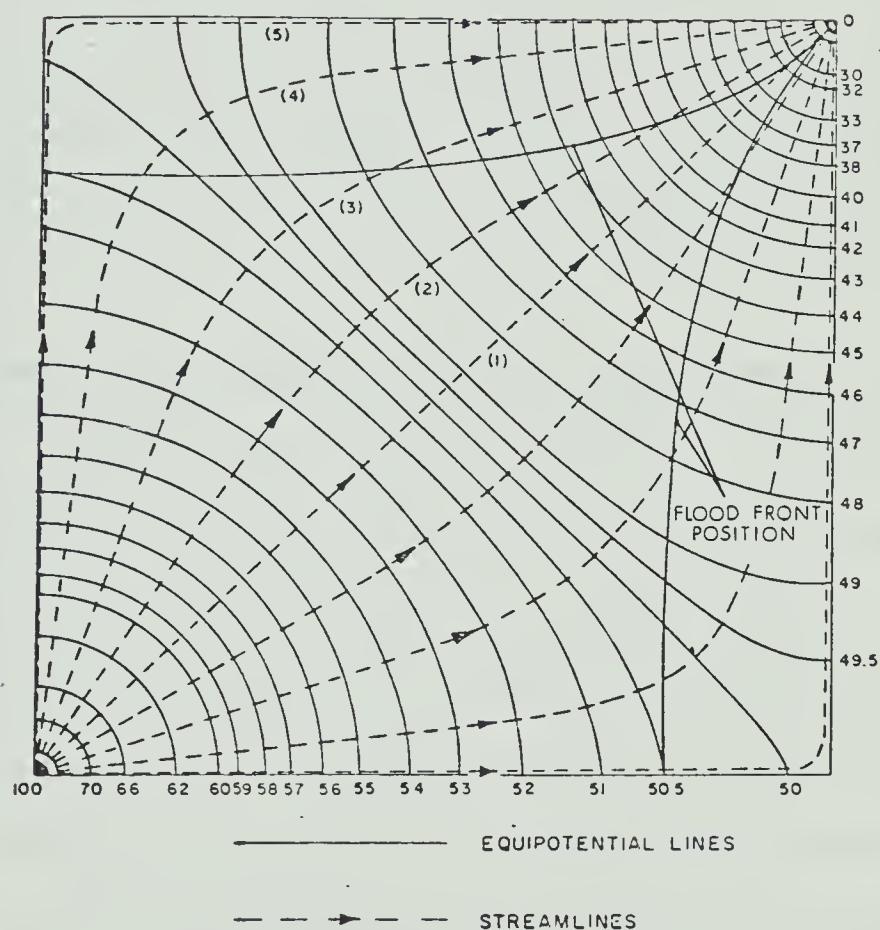


FIGURE - I



and producing wells. Cells along a particular channel would then be of varying length and differing effective cross-sectional area. The resistance to flow because of the shape and size of the cells is represented by  $G$ , the shape factor which is a measure of the geometric resistivity. In equation form, the shape factor is defined as:

$$G = \frac{L}{A} \quad (1)$$

Higgins, Boley, and Leighton (27) have calculated shape factors and channel volumes for the direct line drive, five spot, seven spot, and staggered line drive well patterns.

In the determination of the shape factors for the five spot pattern advantage was taken of its inherent symmetry and only one-eighth of a five spot was used. Each channel was divided into 40 cells of equal volume. This number of cells was chosen on the basis of results of another investigation which indicated that 40 cells were more than a sufficient number to insure good convergence properties during the calculation of a water flooding performance (28).

Where potentiometric data are available on an irregularly shaped well pattern, a careful construction of the flowlines and a division into cells will result in specific values for the shape factor,  $G$ . In the case of



irregularly shaped cells, the shape factor is defined as:

$$G = \frac{L_l + L_r}{L_t + L_b} \quad (2)$$

where  $L$  = length, the subscripts  $l$ ,  $r$ ,  $t$  and  $b$  refer to the left side, right side, top and bottom of the cell and the channel thickness is unity.

If the cells are extremely irregular, Henley (29) has proposed a technique where the shape factor is determined by using inscribed circles. Higgins, et.al. (30) have developed a computer calculation technique whereby volumes of the channels and the shape factors may be determined from data taken from a potentiometric model study for any well spacing pattern.

Kufus and Lynch (31) have shown that the average permeability of a cell times the shape factor determines the resistance to flow. Then, from Darcy's law for a linear horizontal system, the producing rate at the beginning of the flood will be:

$$q_{oj} = \frac{K_a \Delta P}{N} \sum_{i=1}^j \frac{\mu_o}{k_{roIW}} G_i \quad (3)$$

where

$j$  = index of increments since beginning of the flood

$i$  = cell number index



$N$  = total number of cells in each channel

$k_{roIW}$  = permeability to oil at irreducible water saturation.

After  $n$  number of cells have been invaded, the producing rate prior to displacing phase (water) breakthrough is:

$$q_{oj} = \frac{K_a \Delta P}{\sum_{K=1}^n \frac{G_K}{\bar{K}_{rw_k} \frac{\mu_w}{\mu_o}} + \frac{N}{\sum_{i=n+1}^{n+1} \frac{\mu_o}{K_{roIW}} G_i}} \quad (4)*$$

where  $\bar{K}$  = mean permeability in a particular cell.

In order to solve equation (4), it is necessary to determine mean values for the oil and water relative permeabilities in each cell when  $j$  number of cells have been invaded. This is accomplished by calculating the  $f_w$  versus  $S_w$  relationship from the following equation:

$$f_w = \frac{1}{1 + \frac{\frac{1}{K_w}}{\frac{1}{K_o} \frac{\mu_w}{\mu_o}}} \quad (5)$$

The mean permeability of the first cell to water at the instant that water breakthrough occurs at the end of the cell is determined by dividing the slope of the  $f_w$  versus  $S_w$  curve at breakthrough,  $f'_{br}$ , by the area under a curve of the resistance to water flow,  $1/K_{rw}$ , versus the slope,  $f'$ . This area is that up to the value of the

---

\* A derivation of equation (4) is presented in Appendix A.



slope,  $f'$ , corresponding to the total volume behind the flood front. Figure II illustrates these relationships.

When the water front has progressed to the end of the second cell, the mean permeability to water in the first cell is one-half of the  $f'_{br}$  value, divided by the area under the resistance curve, up to the abscissa value of one-half  $f'_{br}$ . The mean permeability to water in the second cell then becomes one half  $f'_{br}$ , divided by the remaining area under the resistance curve. The following equation presents the procedure outlined in an analytical form:

$$\bar{K}_{rw_i} = \frac{\left( \frac{f'_{br}}{j} \right)}{\sum_{\substack{\text{All areas under } 1/K_{rw} \\ \text{vs. } f' \text{ curve in } n^{\text{th}} \\ \text{cell}}} \quad (6)*$$

A similar equation can be readily written for the mean permeability to oil. When breakthrough occurs (the subordinate phase) at the end of one of the channels, equation (4) must be modified to include a water producing rate:

$$(q_o + q_w)j = \frac{K_a \Delta P}{\sum_{i=1}^N \frac{G_i}{\frac{\bar{K}_{rw_i}}{\mu_w} + \frac{\bar{K}_{ro_i}}{\mu_o}}} \quad (7)$$

---

\* A derivation of equation (6) is presented in Appendix A.



WATER SATURATION, RELATIVE PERMEABILITY  
TO WATER, AND RESISTANCE VERSUS  
THE SLOPE TERM,  $f'$  (AFTER HIGGINS-LEIGHTON)

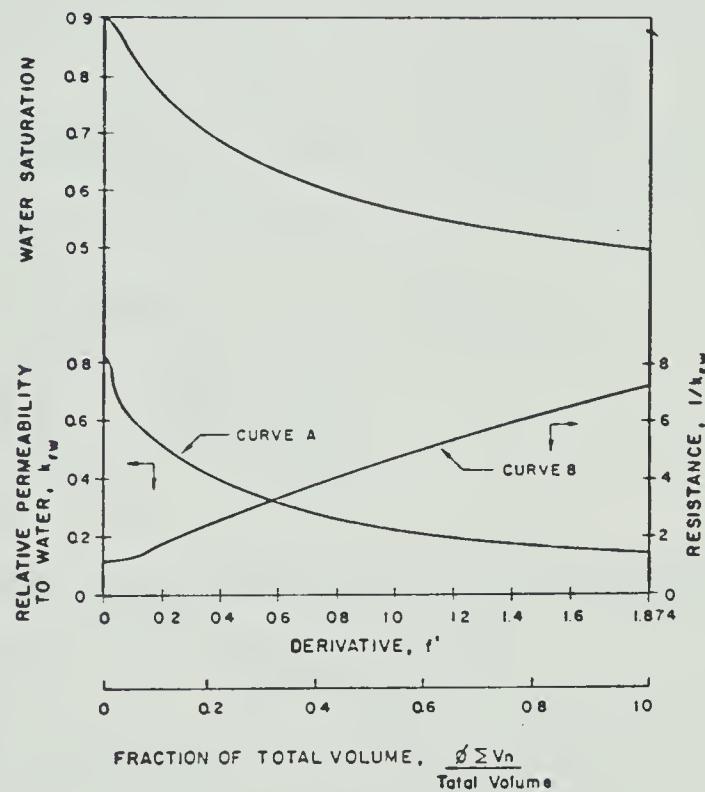


FIGURE - II



The mean permeabilities to water and oil in each cell are now modified due to the decrease of the slope of the  $f'_w$  versus  $S_w$  curve, as water saturations increase beyond that at water breakthrough. In equation form, the mean permeability to water in any given cell becomes:

$$\bar{K}_{rw_i} = \frac{f'_{br-m}}{\sum \text{All areas under } 1/K_{rw} \text{ vs. } f' \text{ curve in } i^{\text{th}} \text{ cell.}} \quad (8)$$

where  $f'_{br-m}$  is the first derivative of the  $f'_w$  versus  $S_w$  relationship from the fractional flow equation at the  $m^{\text{th}}$  increment after water breakthrough. The equation for mean oil permeability takes a similar form. After the  $m^{\text{th}}$  increment of production has occurred past water breakthrough, the water - oil ratio is:

$$(WOR)_j = m+N = \frac{K_{rw}}{\mu_w} \frac{\mu_o}{K_{ro}} \quad (9)$$

The oil and water rates can now be calculated by solving equations (7) and (10) simultaneously.

$$q_{oj} \text{ (subordinate)} = \frac{q_w}{(WOR)_j = m+N} \quad (10)$$

Since the mean permeability values are used to calculate the oil and water producing rates in the subordinate phase, the instantaneous producing rates at the well may be approximated as follows:



$$(q_o)_{j-1/2} = \frac{(q_o)_j + (q_o)_{j+1}}{2} \quad (11)$$

$$(q_w)_{j-1/2} = \frac{(q_w)_j + (q_w)_{j+1}}{2} \quad (12)$$

Here, the index is equal to  $m + N$ , or the  $m^{\text{th}}$  increment of production past water breakthrough, where  $N$  is the total number of cells in each channel.

Since each channel was divided into  $N$  cells of equal volume, the average volume of oil produced from each cell during the primary phase may be written as:

$$v_{ol} = \frac{v_p (s_{w_{Br}} - s_{wIW})}{N} \quad (13)$$

where

$v_p$  = total pore volume of the channel

$s_{w_{Br}}$  = average water saturation behind the front at time of breakthrough.

$s_{wIW}$  = connate or irreducible water saturation.

During the subordinate phase, the Welge equation applies.

The following form of the equation is convenient:

$$\bar{s}_{w_m} = s_{w_m} + \frac{1 - f_{w_m}}{f'_{w_m}} \quad (14)$$

where  $\bar{s}_w$  = average water saturation in a channel, and the index  $m$  refers to the number of increments after initial breakthrough of water at the producing well.



As in a linear calculation of the frontal displacement mechanism, the oil produced during each separate step of the subordinate phase may be calculated as follows:

$$v_{o_j} = m + N = v_p (\bar{s}_{w_{m+1}} - \bar{s}_{w_m}) \quad (15)$$

The time for each step can then be readily determined from the following equation:

$$t_j = \frac{v_{o_j}}{q_{o_j} - \frac{1}{2}} \quad (16)$$

Utilizing equations (1-16) performance is calculated for each channel individually until a preset water-oil ratio is exceeded. The performance of these channels is then combined, using the time for channel 4 as a reference, to get the total production history. The performances of channels 3, 2, and 1 from their ending times to the ending time of the 4th channel are obtained by linear extrapolation.

It may be noted that in this calculation technique, an areal coverage factor has not been used. Such a procedure is not required, since breakthrough at the end of each particular cell in each channel defines the position of the front.



### THE MODIFIED SCHEME

The theory and equations used in this scheme are basically the same as those used by Higgins and Leighton. It may be recalled that the combined performance in the Higgins-Leighton method is obtained by combining the performances of various channels on the same time basis. This introduces an error in the prediction since straight line interpolation and extrapolation is used to calculate the production history at intermediate time intervals. In order to circumvent this problem, all the four channels are flooded simultaneously in the modified approach. This is accomplished by using channel one as a reference and invading complete cells, one at each step, in this channel. The time required to invade a complete cell in channel one is then used to locate the position of the front in the remaining channels.

Although the Higgins-Leighton method assumes that there is no cross flow between various channels, no attempt was made by them to verify this assumption. In attempting to flood the four channels simultaneously in the suggested modification, an iterative scheme was set up to calculate the position of the flood front in channels 4, 3, and 2 relative to channel 1 by using no cross flow between the channels as a convergence criterion.



It may be recalled that Higgins-Leighton method is applicable to only one case where a constant pressure drop is maintained between the injection and the production well. In order to circumvent this limitation, the modified approach considers the following cases:

- constant pressure drop between the injection and the production well
- constant injection rate
- variable injection rate and pressure drop.

#### CONSTANT PRESSURE DROP BETWEEN THE INJECTION AND THE

#### PRODUCTION WELL

##### Primary Phase

In order to prevent cross flow between the various channels, the pressure drop between the injection and the production well at each step in each channel must be identical. However, since the resistance to flow changes as front advances towards the production well, the water rates in each channel must change to account for the corresponding change in resistance. Therefore, the iterative scheme should be such as to account for the variations in water rates with time. Consequently, the following steps were used in implementing the iterative scheme.

1. Before initiating the flood, flow resistances in each channel are computed by using the following equation:



$$R_i = \sum_{K=1}^N \frac{\mu_o G_k}{K_{roIW}} \quad (17)$$

where  $i$  denotes the channel number

2. These flow resistances are then used to calculate initial water rates in the various channels, using the following relationship:

$$q_i = \frac{\Delta P \times K_a}{R_i} \quad (18)$$

$i = 1 \rightarrow 4$

3. This information is then used to calculate the time step required to invade one complete cell in channel 1. This has been accomplished by using the following scheme:

$$\text{Volume of oil produced when one complete cell is invaded} = \frac{V_p (S_w_{BR} - S_w_{IW})}{NCELLS} \quad (19)$$

where

$NCELLS$  = total number of cells in a channel,

$V_p$  = pore volume of a channel,

$S_w_{BR}$  = average water saturation behind the front at the time of breakthrough, and

$S_w_{IW}$  = connate or irreducible water saturation.

Assuming that the flow rate calculated in step 2 remains constant for this time period, the volume of water injected would then be equal to  $q_i \times \Delta T_i$ , where  $\Delta T_i$  is the time step.



In the primary phase since the volume of oil produced is equal to the volume of water injected,

$$\Delta T_i = \frac{V_p (S_w_{BR} - S_w_{IW})}{q_i \times NCELLS} \quad (20)$$

4. Using this time step, the flood front positions in channels 2, 3, and 4 are determined by using the following equation:

(21)

$$\text{No. of cells invaded (in channels 2,3,4)} = \frac{\text{Avg. w.r.} \times \Delta T_i \times NCELLS}{V_p \times (S_w_{BR} - S_w_{IW})}$$

5. Once the flood has been initiated and the first cell in channel 1 has been invaded, oil and water both flow behind the front, whereas only oil flows ahead of the front. Consequently, the total resistance to flow in each channel is the sum of the resistance due to two-phase flow and that due to single-phase flow. This is represented by the following equation:

$$R_i = \frac{n}{\sum_{j=1}^n \frac{G_j}{\frac{\bar{K}_{ro,j}}{\mu_o} + \frac{\bar{K}_{rw,j}}{\mu_w}}} + \frac{N}{\sum_{j=n+1}^{n+1} \frac{\mu_o}{\bar{K}_{roIW}} G_j} \quad (22)$$

where n = number of cells invaded.



6. Using the flow rates calculated in step 2 as an initial approximation, the pressure drops in each channel are calculated according to equation (18). These pressure drops are then compared against the one read into the computer initially. If the two values differ more than that allowable by the error criterion, the flow rates in each channel are modified using the following equation:

$$q_{i \text{ new}} = q_{i \text{ previous}} \times \frac{\Delta P_{\text{original}}}{\Delta P_{\text{calculated}}} \quad (23)$$

$i = 1 \rightarrow 4$

7. Steps 3 to 6 (inclusive) are repeated until the error criterion is satisfied.
8. Steps 1 to 7 (inclusive) are repeated until the 40 cells in channel 1 are invaded.

#### Subordinate Phase

After breakthrough it is believed that the average water saturation behind the flood front gradually increases and that the derivative  $\frac{df_w}{ds_w}$  gradually decreases. In order to simulate these conditions in the modified version  $\frac{df_w}{ds_w}$  was decreased in steps according to the following equation:

$$NSEND = \frac{NEND \times 40}{(40 + \text{additional steps})} \quad (24)$$



where

NEND = Total number of divisions in f'grid used  
to calculate mean permeabilities in the  
various cells, and

NSEND = Total number of divisions remaining in  
f'grid at any step.

This information is then used to compute the  
water-oil ratio according to the procedure outlined in  
the Higgins-Leighton method.

The nature of the channels is such that the  
resistance to flow is greatest in channel 4 and decreases  
in order of magnitude from channel 4 to channel 1. As  
a result when channel 1 has broken through channels 2,  
3, and 4 are only partly invaded.

Since in the modified approach all the four  
channels are flooded simultaneously and different schemes  
are used for treating primary and subordinate production,  
it is necessary to monitor the position of the flood  
front in each channel. After channel 1 has broken through,  
the position of the flood front in the remaining channels  
is observed in order to determine when to use the subor-  
dinate phase calculations in other channels.

Finally, the flood is terminated when the com-  
bined instantaneous water-oil ratio from all the four  
channels exceeds a preselected value.



## CONSTANT INJECTION RATE

### Primary Phase

In order to maintain a constant total injection rate, and also to satisfy the no cross flow criterion, the following conditions must be fulfilled:

- the sum of the injection rates of the various channels at each step must equal the total injection rate read initially into the computer; and,
- to satisfy the no cross flow criterion, pressure drop between the injection and the production well at each time step, in each channel, must be identical. In view of equation (18), this can be stated mathematically in terms of the expression:

$$\frac{q_1 R_1}{K_a} = \frac{q_2 R_2}{K_a} = \frac{q_3 R_3}{K_a} = \frac{q_4 R_4}{K_a} = \frac{qR}{K_a} \quad (25)$$

or

$$q_1 R_1 = q_2 R_2 = q_3 R_3 = q_4 R_4 = qR \quad (26)$$

In order that these requirements be satisfied, the iterative scheme employed consists of the following steps:

1. First the initial resistance to flow in each channel is calculated using equation (17).
2. The initial water rates in all the channels are then determined using equation (18).
3. As soon as the first cell in channel 1 is invaded,



the time step and the position of the flood front in the remaining channels is computed using equations (20) and (21).

4. Next, resistances in all channels are determined using the position of the flood front in each channel as the boundary between the two-phase and the single-phase region. Equation (22) is used for this purpose.
5. The average of the product of flow rate and resistance in all the four channels is then calculated by means of the following equation:

$$\text{PROD} = \left[ \sum_{i=1}^4 q_i R_i \right] / 4.0 \quad (27)$$

6. Following this, a check is made to determine whether or not the product of flow rate and resistance in each channel agrees with PROD within the limits of a chosen error criterion. The following equation is used in making this comparison:

$$\text{CHECK} = \left| \frac{(q_i R_i - \text{PROD})}{\text{PROD}} \right| \quad (28)$$

7. If the magnitude of CHECK exceeds the error criterion, the flow rates in each channel are modified using the following equation:

$$q_{i \text{ new}}^{th \text{ channel}} = q_i \text{ previous} - \frac{\left( q_i \text{ previous} \times R_i - \text{PROD} \right)}{\text{PROD}} \quad (29)$$



8. The injection rates in all the channels are then added together according to equation:

$$\text{SUM} = \sum_{i=1}^4 q_i \quad (30)$$

The quantity SUM is then compared to the total injection rate,  $Q_T$ , which was read into the computer at the start of the program. This comparison is made through the use of the equation:

$$\text{RERROR} = \left| \frac{\text{SUM} - Q_T}{Q_T} \right| \quad (31)$$

9. If the magnitude of RERROR, is greater than some preselected value, the flow rates in each channel are modified by using the following equation and,

$$q_{i_{\text{new}}} = q_{i_{\text{previous}}} \times \frac{Q_T}{\text{SUM}} \quad (32)$$

steps 3 to 8 (inclusive) are repeated.

10. Once the conditions CHECK and RERROR are satisfied, the next cell in channel 1 is invaded.
11. Steps 3 to 10 (inclusive) are then repeated until 40 cells in channel 1 have been invaded.
12. Once channel 1 has broken through the position of the flood front in the remaining channels is monitored as in the constant pressure drop case.



Finally, the flood is terminated when the combined instantaneous water-oil ratio from all the four channels exceeds a preselected value.

#### VARYING INJECTION RATE AND PRESSURE DROP

The iterative scheme in this case is identical to the one employed in the constant injection rate case. However, since the total injection rate,  $Q_T$ , may change with time, an interpolation scheme is required to determine this injection rate for cumulative times calculated at each step from time versus injection rate history. This value of injection rate determined at each step is then used in place of  $Q_T$  in the iterative scheme described for the constant injection rate case.

A detailed listing of the computer program used in the modified version is presented in Appendix B.

#### RECENT WORK OF HIGGINS

Subsequent to the development of the modifications just described Higgins presented (32) a method which extends the Higgins-Leighton method to cover the constant rate case. His approach seems to differ considerably from the one suggested in the present investigation. Although most of the working details are not obvious from the pre-print available at the time of writing this report, the following points can still be emphasized to indicate the differences involved.



1. The production history for the constant pressure drop case is a pre-requisite for the calculation of the production history for constant rate case.
2. The instantaneous oil rate is obtained by dividing the constant injection rate by the instantaneous injection rate under constant pressure and multiplying the quotient by the instantaneous oil rate computed in the constant pressure drop case.
3. The time steps for the constant injection rate case are obtained by multiplying the fractional pore volume of water injected by the number of barrels of pore volume in the five spot pattern, and then dividing the product by the constant injection rate. It may be noted that this is on the basis of the total pattern and not any individual channel which is the case in the modified scheme.
4. The method does not use an iterative scheme and no attempt has been made to check whether the instantaneous rate used in a channel at a particular time step complies with the no cross flow assumption.

Due to limitations of time a detailed examination of Higgins modification (33) has not been conducted. Consequently, his method and the one reported herein have not been compared.



### RESULTS AND DISCUSSION

The modified scheme was applied to a five spot pattern studied experimentally by Douglas, Peaceman, Rachford (34) and theoretically by Higgins and Leighton (35). The results obtained are tabulated in Appendix C and compared in Figures III and IV.

Figure III presents a variation in pore volumes of oil produced as a function of pore volumes of water injected. It may be observed that until water injected exceeds 2.0 pore volumes, the results obtained from the modified approach are closer to the laboratory data as compared to the Higgins-Leighton results. It may also be noted that the modified scheme yields identical results in both cases where injection rate or pressure drop is maintained constant. This can be explained on the basis that since maintaining the injection rate or pressure drop constant affects only the time rate of oil production, the results must be compared against the time parameter.

Figure IV presents a variation in instantaneous water-oil ratio as a function of time. It can be clearly seen that the modified approach yields different results for both cases where injection rate or pressure drop is maintained constant. Since, instantaneous water-oil ratio versus time history was not reported by Douglas et.al. (36), the results of the modified approach could



PLOT OF OIL RECOVERY VS PORE-VOLUME INJECTED

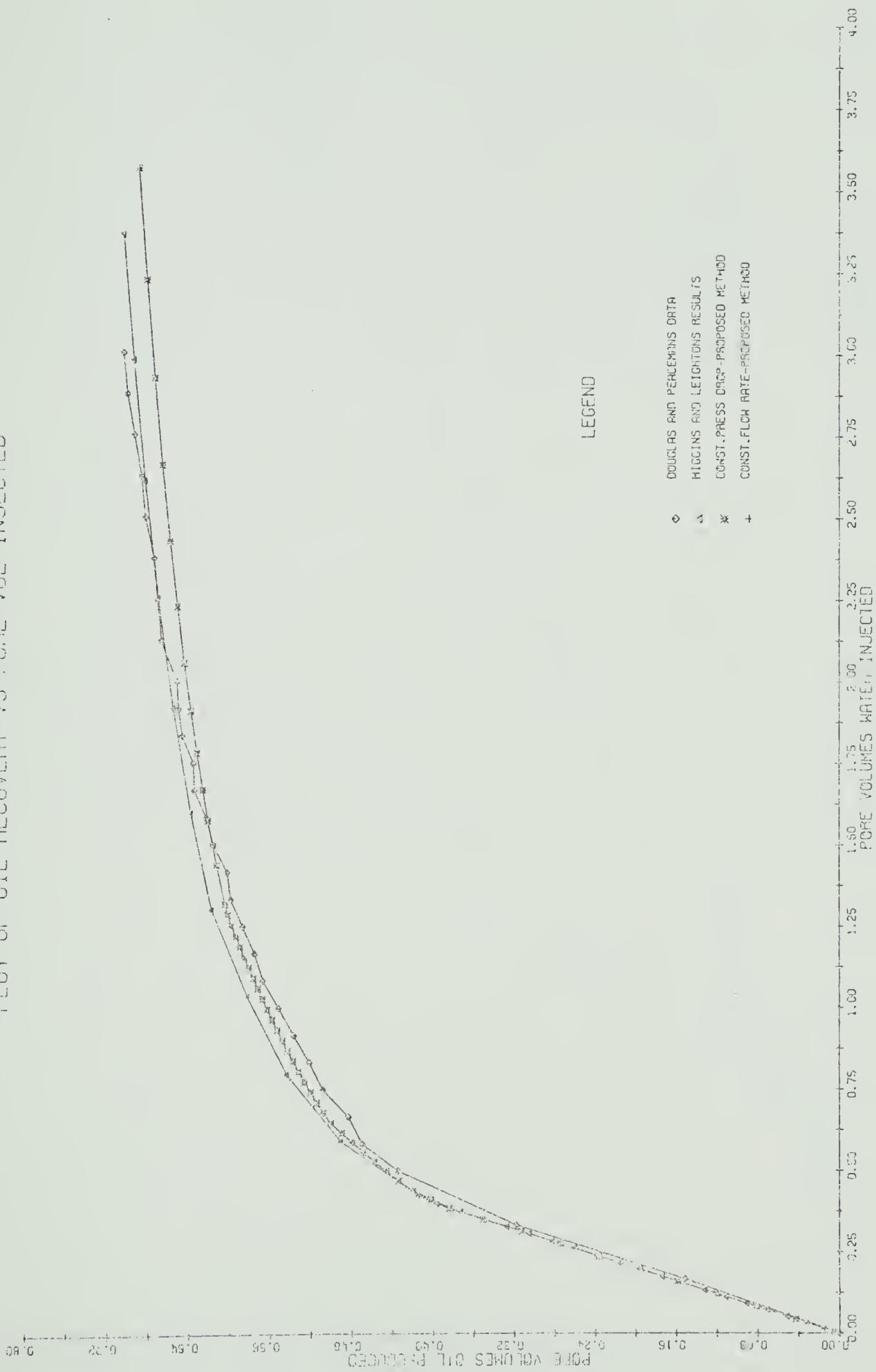


Figure III



PLOT OF TIME VS WATER-OIL-RATIO

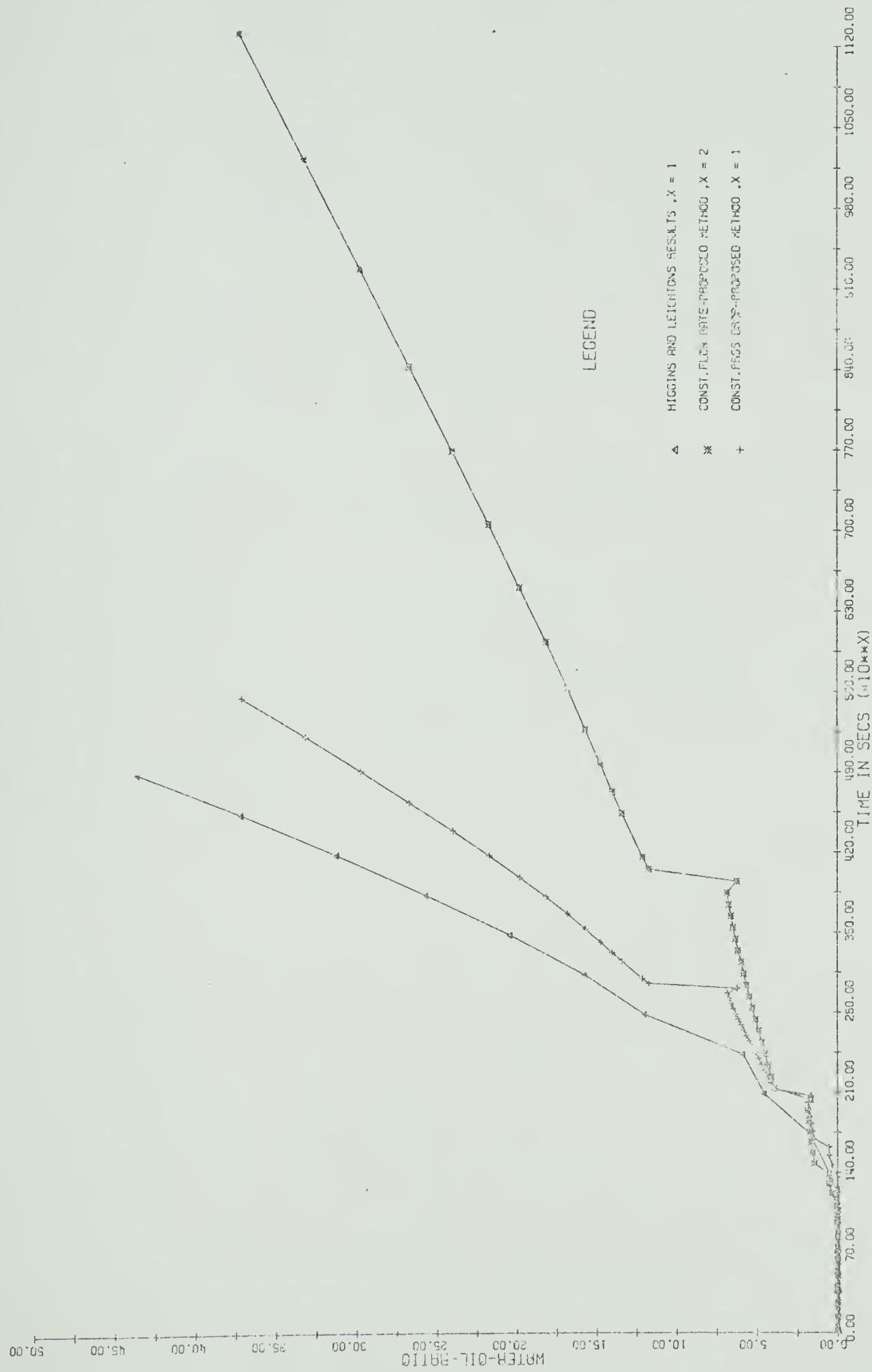


Figure IV



not be compared against the laboratory data. However, a comparison with Higgins-Leighton results is presented and reveals that there are discontinuities in the results obtained from the modified approach. These discontinuities represent breakthrough of each channel. Since the results obtained from the Higgins-Leighton method were averaged before plotting, these discontinuities are, therefore, absent in the data calculated from their method.

The modified scheme requires relative permeability versus saturation curves as its basic data. Within the limitations of time provided for conducting the present investigation representative relative permeability versus saturation data for the laboratory model used in the present investigation could not be obtained and thus the attempts at checking compatibility of laboratory results proved in vain.



## CONCLUSIONS

The conclusions drawn as a result of this study are:

1. The Higgins-Leighton method has been modified and extended to predict the performance of water floods for the following cases:
  - constant injection rate
  - constant pressure drop between the injection and the production well
  - variable injection rate and pressure drop.
2. The modified approach was tested on the laboratory data reported by Douglas, et.al. (37). The agreement obtained between the experimental and calculated results was well within the desired degree of accuracy.
3. The use of an iteration scheme in the proposed method ensures that no cross flow occurs between the various channels. Thus, although the shape factors used are for unit mobility ratio, possibilities of obtaining better correspondence between laboratory results and calculated prediction for adverse mobility ratios look favourable.



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## APPENDIX A

- Derivation of the equation used to calculate oil rate in the primary phase after first cell has been invaded.
- Derivation of the equation to determine the mean permeability in a cell.



EQUATION TO COMPUTE OIL RATE IN THE PRIMARY PHASE AFTER  
FIRST CELL INVASION

By Darcy's Law for horizontal flow

$$Q = \frac{-K_a K_r \Delta P}{\mu L/A} \quad (1)$$

If one defines the shape factor

$$G = L/A$$

than

$$Q = \frac{-K_a K_r \Delta P}{\mu G} \quad (2)$$

$$= -\frac{\Delta P}{\mu G}$$

$$\frac{K_a K_r}{}$$

By analogy to Ohm's Law one may write this equation as:

$$Q = \frac{-\Delta P}{R} \quad (3)$$

where  $R = \frac{\mu G}{K_a K_r}$  and is called the resistance to flow. Thus for a simultaneous flow of oil and water, the resistance to flow for each phase would be given by:

$$R_{\text{oil phase}} = \frac{\mu_o G}{K_a K_{r_o}} \quad (4)$$

$$R_{\text{water phase}} = \frac{\mu_w G}{K_a K_{r_w}} \quad (5)$$



After the invasion of the first cell, there is two-phase flow behind the flood front in the first cell consisting of oil and water, and single-phase flow ahead of the front consisting only of oil. Therefore, the total resistance to fluid flow will comprise

$$R_{\text{total}} = R_{\text{two-phase}} + R_{\text{single-phase}}$$

where

$$R_{\text{two-phase}} = \frac{G}{K_a \left[ \frac{K_r_w}{\mu_w} + \frac{K_r_o}{\mu_o} \right]}, \text{ and}$$

$$R_{\text{single-phase}} = \frac{G}{K_a K_{r_{OIW}} \mu_o}$$

Thus,

$$R_{\text{total}} = \sum_{i=0}^{i=1} \frac{G_i}{K_a \left[ \frac{K_r_o}{\mu_o} + \frac{K_r_w}{\mu_w} \right]} + \sum_{j=i+1}^N \frac{\mu_o G_j}{K_a K_{r_{OIW}}}$$
(6)

where,

$K_{r_{OIW}}$  = relative permeability to oil at irreducible water saturation.

Substituting this value of  $R_{\text{total}}$  in equation (3) yields:

$$Q_{\text{obj}} (\text{primary}) = \frac{-\Delta P K_a}{\sum_{i=0}^1 \frac{G_i}{\frac{K_{r_o i}}{\mu_o} + \frac{K_{r_w i}}{\mu_w}}} + \sum_{j=i+1}^N \frac{\mu_o G_j}{K_{r_{OIW}}}$$



con't. .

$$= \frac{-K_a \Delta P}{\sum_{i=1}^n \frac{G_i}{\frac{\mu_o}{K_{ro,i}} + \frac{1}{K_{rw,i}}} + \sum_{j=n+1}^N \frac{\mu_o G_j}{K_{roIW}}} \quad (7)$$

### EQUATION TO DETERMINE THE MEAN PERMEABILITY IN A CELL

The resistance to flow of fluids in a linear, horizontal system is described by the following equation:

$$q = \frac{-\Delta P}{R} \quad (1)$$

where  $R = \frac{G}{K_a K_r}$  and is termed as the resistance.

Realizing that this resistance,  $R$ , varies with the position of the flood front, it is convenient to define an average value  $\bar{R}$  such that,

$$\bar{R} = \frac{\int_0^L R dx}{\int_0^L dx} \quad (2)$$

but

$$R = \frac{\mu G}{K_a K_r}$$

$$\therefore \bar{R} = \frac{\int_0^L \frac{\mu G}{K_a K_r} dx}{\int_0^L dx}$$

However, for the problem at hand  $\mu$ ,  $G$ , and  $K_a$  are independent of  $X$ .



$$\therefore \bar{R} = \frac{\frac{\mu G}{K_a}}{\frac{\int_0^L \frac{dx}{Kr}}{\int_0^L dx}} \quad (3)$$

However, one may also write,

$$\bar{R} = \frac{\mu G}{Ra Kr}$$

but,  $\mu_o$ ,  $G$ , and  $K_a$  remain constant

$$\therefore \bar{R} = \frac{\mu G}{K_a Kr} \quad (4)$$

so that,

$$\bar{Kr} = \frac{\mu G}{K_a \bar{R}} \quad (5)$$

Substituting for  $\bar{R}$  from equation (3) one obtains

$$\begin{aligned} \bar{Kr} &= \frac{\mu G}{K_a} \times \frac{K_a}{\mu G} \times \frac{\int_0^L dx}{\int_0^L \frac{dx}{Kr}} \\ \therefore \bar{Kr} &= \frac{\int_0^L dx}{\int_0^L \frac{dx}{Kr}} \end{aligned} \quad (6)$$

Using Buckley-Leverett's material balance equation

$$\begin{aligned} x &= \frac{qt}{\phi A} \frac{df_w}{ds_w} \\ &= \frac{qt}{\phi A} f'_w \end{aligned} \quad (7)$$



Differentiating both sides

$$dx = \frac{qt}{\phi A} df'_{w} \quad (8)$$

(Assuming  $qt$ ,  $\phi$ , and  $A$  are constant)

Substituting equation (8) in equation (6) one obtains

$$\overline{Kr} = \frac{qt}{\phi A} x \frac{\phi A}{qt} \frac{\int_0^L df'_{w}}{\int_0^L \frac{1}{Kr} df'_{w}}$$

Simplifying

$$\overline{Kr} = \frac{\int_0^L df'_{w}}{\int_0^L \frac{1}{Kr} df'_{w}} \quad (9)$$

Applying boundary conditions,

$$\text{At } x = 0 ; \quad f'_{w} = 0$$

$$x = L ; \quad f'_{w} = f'_{w_{br}}$$

$$\therefore Kr_{\text{mean}} = \frac{f'_{w}}{\int_{w_{br}}^{br} \frac{1}{Kr} df'_{w_{br}}} \quad (10)$$



Thus, for oil and water phases this equation will take the following form:

$$K_{ro\text{mean}} = \frac{\int_0^f w_{br} \frac{1}{Kr_o} df}{\int_0^f w_{br} df} \quad (11)$$

$$K_{rw\text{mean}} = \frac{\int_0^f w_{br} \frac{1}{Kr_w} df}{\int_0^f w_{br} df} \quad (12)$$



## APPENDIX - B

- Basic data required to use the method
- Nomenclature for computer program
- Listing of the computer program



BASIC DATA REQUIRED TO USE THE METHOD

The following data are required to predict the performance of any pattern using the modified scheme:

1. Relative permeability versus saturation curves.
2. Pore volumes and shape factors for each channel.
3. Initial water saturation (connate or irreducible) and irreducible oil saturation.
4. Maximum water - oil ratio at which cut-off is desired.
5. Pressure drop between the injection and the production well for a constant pressure drop case.
6. Injection rate for a constant injection rate case.
7. Time versus injection rate history where both injection rate and pressure drop vary.
8. Absolute permeability of the medium.
9. Oil and water viscosities.



NOMENCLATURE FOR COMPUTER PROGRAM

ALPA ( )	= Left part of a unit area, $\Delta f'$ times reciprocal of permeability.
ALPAPT	= Left part on the base of a unit, $\Delta f'$ .
AVGSAT	= Average saturation behind the flood front after breakthrough
AVPERM	= Name of subroutine to determine the average permeability to water.
AVPER2	= Name of subroutine to determine the average permeability to water for fractional cells.
D1 ( )	= First difference.
D2 ( )	= Second difference.
D3 ( )	= Third difference.
D11 ( )	=
D22 ( )	= Same as above but for water permeability.
D33 ( )	=
DEL	= Width of an interval into which the saturation range between irreducible water and oil is divided; used for determining $s_w$ at breakthrough.
DEL2	= Width of an interval into which the saturation range between $f'$ at breakthrough and irreducible oil is divided.
DEL3	= Width of a division into which range of $f'$ between $f'$ at breakthrough and $f'$ at the irreducible oil saturation is divided.
DELN	= Number of divisions into which the water saturation range between the interstitial water and irreducible oil saturation is to be divided in order to attain the $f$ function and the derivative.
DJ	= Decimal expression of J.



DN = Decimal expression of N.  
 DNCELL = Decimal expression for the integer NCELLS.  
 DNDIVS = Decimal expression of NDIVS.  
 DS = Number of divisions between  $S_w$  at breakthrough  
       and  $S_w$  at irreducible oil.  
 DSMN1 = Decimal expression of NDSMN1.  
 ELTIME = Total elapsed time.  
 ENDPO = Last permeability to oil in the permeability  
       lookup table; irreducible oil.  
 ENDPW = Last permeability to water in the permeability  
       lookup table; irreducible oil  
 ENDSW = Water saturation at irreducible oil, same  
       value as SWIO.  
 F ( ) = Fractional flow of water equals  $1/(1 + k_{ro} \mu_w / k_{rw} \mu_o) = f_w$ .  
 FA ( ) = Average of two  $f_w$  values.  
 FCELL ( ) = Retains the value of FRCELL ( ).  
 FACTOR = Proportionality factor to interpolate for  
       saturation at breakthrough.  
 FB =  $f_w$  at breakthrough adjusted by interpolation.  
 FLORTE ( ) = Subroutine used to interpolate for QT in a  
       case where both pressure drop and Injection  
       rate vary.  
 FLOCHK ( ) = Subroutine used to check cross flow.  
 FP ( ) = Derivative of  $f_w$ .  
 FPB =  $f'$  at breakthrough.  
 FPP ( ) =  $f'$ ; it is the same as FP but the computer  
       program requires a double indexing during  
       part of the calculation.  
 FRCELL ( ) = Total number of cells invaded including  
       fractional cells.



G ( ) = Shape factor,  $L/A$ , which is a function of the geometry of the system.  
 GEMTRY = Subroutine for geometry - shape factors and volumes - of the system.  
 GGG = Single shape factor used for linear case only.  
 H = Difference between succeeding X values; used in STIRLG subroutine.  
 I = Index for channel number.  
 IGEOM = Test if linear or nonlinear geometrical shape factors are to be read in or previous shape factors to be reread and used. If zero, values from previous case are used, if 1, nonlinear values are read, if 2, linear value is used.  
 II = Tally for channel number.  
 IPA VPM = Print out in the AVPERM mean permeabilities.  
 IPSETC = Print out  $S_w$ 's,  $k_{rw}$ 's,  $k_{ro}$ 's,  $f_w$ 's, and  $f_r$ 's, at selected interval. | If zero, do not print; if positive, print.  
 IST = Number of channels.  
 J = Index.  
 JJ = Index.  
 JPNTM1 = JPOINT - 1.  
 JPNTM2 = JPOINT - 2.  
 JPNTM3 = JPOINT - 3.  
 JPOINT = Ending index of permeability data points taken from empirical curve.  
 K = Index.  
 KABS = Absolute permeability.  
 KCASE = Index to indicate if another case is to follow.



KOUNT = Number of iterations required for convergence.

L = Index.

LEND = Retains L for later indexing.

LFPND = GO TO index, first sets index of saturation distribution to end and then reduces the end by NSTEPS. If 1, set to end; if 2, reduce by NSTEPS.

M = Index.

N = Index.

NCELLS = Number of cells into which a channel is divided.

NDELL = Integer part of FRCELL ( ).

NDIVS = Number of divisions (cell length) into which piston is to be divided.

NDL = Used to read in only once the points for the permeability lookup curve.

NDS = Integer for decimal DS; number of divisions of saturation range between irreducible oil and breakthrough.

NDSM1 = NDS - 1.

NDSP1 = NDS + 1.

NDVSM1 = NDIVS - 1.

NEND = Ending index for number of units used in AVPERM subroutine.

NENDM1 = NEND - 1, used for summing PAW's.

NIN = Integer equivalent to decimal DELN, or number of saturation increments between, SWIW, and SWIO for determining breakthrough.

NINM1 = NIN - 1.

NINP1 = NIN + 1.

NN = Index.



NNQ = Index to determine how many channels have been invaded.

NSEND = Number of divisions in f'grid for complete cells invaded.

NSPCE = Number of indices between printing of saturation distribution values.

NSWBR = Index routing for loose, tight, or normal piston. If 1, use Welge's equation; if 2, use trapezoidal equation.

NT = Index for finding ends and beginning of the units in the subroutine, AVPERM.

NTM1 = NT - 1.

NTTP = NT retained before advancing again on NT.

NTTP1 = NTTP + 1.

OILRAT ( ) = Oil rate from Ith channel at Nth time.

OILRCV ( ) = Oil recovered from a channel.

PA ( ) = Average of two reciprocals of permeability; used in AVPERM.

PABR ( ) = Mean permeability determined from mean reciprocal permeability in AVPERM subroutine.

PABRL ( ) = Reciprocal of average permeability used in AVPERM.

PABRO ( ) = Mean permeability to oil in a cell,  $k_{ro_{mean}}$ .

PABRW ( ) = Mean permeability to water in a cell,  $k_{rw_{mean}}$ .

PALE = Pore volume of a channel,  $V_p$ .

PAM ( ) = Average reciprocal permeability in one of the units into which reciprocal permeability of the piston is divided.

PAO = Average reciprocal permeability to oil of a unit.



PAW ( ) = Average reciprocal permeability to water of a unit.  
 PBWN ( ) = Reciprocal of permeability at cell boundaries; used for determining mean permeability at high water - oil ratios.  
 PERMA = Subroutine for calculating permeabilities.  
 PO ( ) = Permeability to oil,  $k_{ro}$ .  
 POIW = Oil permeability at irreducible water saturation  
 $k_{ro}_{IW}$ .  
 PRDCAL = Calculated pressure drop between the injection and production well.  
 PRDRP ( ) = Total pressure drop at each step.  
 PRSS = Pressure drop across entire channel times the absolute permeability; from input to output well,  $\Delta P \times K_a$ .  
 PVINJ = Pore volumes injected in Ith channel at Nth time.  
 PW ( ) = Permeability to water,  $k_{rw}$ .  
 PWC = Permeability to water at connate water saturation,  $k_{rw_c}$ .  
 Q ( ) = Rate of flow in a channel.  
 QT( ) = Total injection rate.  
 RCELL = Fractional cell invaded in Ith channel at any step.  
 RERROR = Relative error between the injection rate and sum of flow rates in all the channels.  
 R ( ) = Resistance to flow in a channel at Ith step.  
 RP ( ) = Reciprocal of permeability; used in AVPERM subroutine.  
 RPA ( ) = Right part of a unit.



RPAPT = Right proportional part of a unit along the  
 f' abscissa.  
 RPO ( ) = Reciprocal of permeability to oil.  
 RPW ( ) = Reciprocal of permeability to water.  
 RSPWM = Retain value of the past sum of divisions.  
 SAT = Difference between breakthrough and connate;  
 water saturation.  
 SAV ( ) = Average water saturation,  $\bar{S}_w$ .  
  
 SGMNT ( ) = Segment, length of intercept on the  $S_w$   
 abscissa made by tangent f', used  
 to determine  $\bar{S}_{w_{br}}$ .  
 SGMPRL = Value of SGMNT for previous division, used  
 for interpolating.  
 SMUNIT ( ) = Cumulative sum of widths of divisions =  $\Sigma$  uts.  
 SPWNDM = Same as WDIV, used to retain value of WDIV  
 when WDIV is changed.  
 SR = Sum of resistances,  $\Sigma R$ .  
 STIRLG = Stirling subroutine name.  
 SUMDIV = Cumulative sum of widths of cell, or distance  
 from input well.  
 SUMG = Summing terms for G'S of invaded cells in  
 a channel,  $\Sigma G$ .  
 SW ( ) = Water saturation,  $S_w$ .  
 SWB = Water saturation,  $S_{w_{br}}$ .  
  
 SWBR = Average water saturation at breakthrough,  
 $\bar{S}_{w_{br}}$ .  
 SWC = Connate water saturation,  $S_{w_c}$ .



SWIO = Water saturation at irreducible oil saturation,  $S_w^{IO}$ .

SWIW = Irreducible water saturation,  $S_w^{IW}$ .

SWW ( ) = Water saturation for second indexing.

T ( ) = Time, t.

TIME ( ) = Instantaneous time for 1st channel at each step.

U = Term used in Stirling equation.

UNITS = Length of division of f' used in calculating average areas for mean permeability.

UO = Viscosity of oil,  $\mu_o$ .

UPA = Term used in Newton forward equation, in Stirling (STIRLG) subroutine.

UPB = Term used in Newton backward equation, in Stirling subroutine.

UW = Viscosity of water,  $\mu_w$ .

UWUO = Viscosity ratio of water to oil,  $\mu_w/\mu_o$ .

VINAL = Initial volume of oil.

VO ( ) = Total volume of oil produced during an increment.

VOO = Total volume of oil produced at each step in channel 1.

WATRAT ( ) = Water rate from Ith channel at Nth time,  $q_w_{j-1/2}$ .

WDIV = Width of a division in the AVPERM.

WOR ( ) = Water - oil ratio.

WORMAX = Maximum water - oil ratio.



WORTIO ( ) = Water - oil ratio for Ith channel at Nth time.

X ( ) = Abscissa in permeability lookup table.

XX = Interpolated X, not on the coincidence values.

YY = Interpolated Y, in permeability lookup.

YO ( ) = Ordinate Y for oil in permeability lookup.

YW ( ) = Ordinate Y for water in permeability lookup.



MAIN

MAIN

C THIS PROGRAM CALCULATES TWO PHASE FLOW IN ANY  
C IRREGULARLY BOUNDED POROUS MEDIUM FOR THE FOLLOWING  
C CASES

- C - CONSTANT PRESSURE DROP BETWEEN THE INJECTION AND  
C THE PRODUCTION WELL
- C - CONSTANT INJECTION RATE
- C - VARYING INJECTION RATE AND PRESSURE DROP

```
COMMON SW(510),SWW(510),PW(510),PO(510),RPW(510),
1RPO(510),PAW(510),PAO(510),PABRW(510),PABRO(510),
1F(510),FP(510),FPP(510),G(4,40),Q(4,500),TIME(500),
1R(4,500),FRCELL(4),VCELL(4),FCELL(4),QT(500),
1PRDRP(500)
COMMON SAT,NCELLS,UO,UW,POIW,UNITS,NEND,IPAVP 1,
1NDL,KOUNT,KCASE,COMTIM,MNQ
DIMENSION PALE(4),WOR(4,500),SR(500),OILRCV(500),
1WORTIO(500),PVINJ(500),VO(500),VOO(500),WATRAT(500),
1OILRAT(500),T(200)
REAL KABS
```

C INITIATION AND DETERMINAION OF BREAKTHROUGH

```
NN = 0
SGMNT = 0.0
PRSS = 100.0
KABS = 11.1
MNQ = 0
NDL = 0
WRITE(6,2009)
```

```
2009 FORMAT('0!',' PERFORMANCE IN PRIMARY RECOVERY ')
READ(5,1001)POIW,UO,UW,IST,KCASE
1001 FORMAT(3E12.5,I6,I6)
READ(5,1002)SWIO,SWIW,SWC,PWC,(PALE(I),I = 1,IST)
1002 FORMAT(4E12.5,4F6.0)
READ(5,1003)DELN,DS,NQREG,NSTEPS,NSPCE,NCELLS,IPAVPM
1003 FORMAT(2E12.5,5I5)
DO 3 I = 1,IST
READ(5,1004)(G(I,N),N = 1,NCELLS)
1004 FORMAT(6E12.5)
3 CONTINUE
UWUO = UW/UO
DEL = (SWIO - SWIW)/DELN
NIN = DELN
NINP1 = NIN + 1
NINM1 = NIN - 1
SW(1) = SWIW
```



## MAIN ... (CONT'D)

```

SW(NINP1) = SWIO
DO 20 N = 2,NIN
SW(N) = DEL + SW(N-1)
CALL PERMA(N,SW,PO,PW,NDL)
20 F(N) = 1.0/(PO(N)/PW(N)*UWUO + 1.0)
F(1) = 0.0
F(NINP1) = 1.0
DO 40 N = 1,NINM1
K = NINP1 + 1 - N
SGMRPL = SGMNT
FP(K) = (F(K) - F(K-1))/DEL
FA = (F(K) + F(K-1))/2.0
SGMNT = FA/FP(K)
IF(SGMNT-(SW(K)-(SWIW+DEL/2.0)))45,45,40
40 CONTINUE
WRITE(6,4002)K
4002 FORMAT('0',' LOOSE PISTON K = ',I6)
GO TO 990
45 IF(K = NINP1)50,48,48
48 WRITE(6,4802)K
4802 FORMAT('0',' TIGHT PISTON K = ',I6)
GO TO 990
50 FACTOR = (SW(K)-SWIW-DEL/2.0-SGMNT)/(SW(K)-SWIW-
1DEL/2.0-SGMNT+SGMRPL-(SW(K+1)-DEL/2.0-SWIW))
FPB = (FP(K+1)-FP(K))*FACTOR+FP(K)
SWB = (SW(K+1)-SW(K))*FACTOR+SW(K)-DEL/2.0
N = 505
SW(N) = SWB
CALL PERMA(N,SW,PO,PW,NDL)
FB = 1.0/(PO(N)/PW(N)*UWUO + 1.0)
SWBR = (1.0 - FB)/FPB + SWB
WRITE(6,5005)FPB,FB,SWB,SWBR,UWUO,K
5005 FORMAT('1','SLOPE = ',E12.5,' F AT TANGENCY = ',
1E12.5,'SATURATION AT TANGENCY = ',F10.5/' EST
1SATURATION AT BREAKTHROUGH = ',F10.5,'UW/UO = ',
1E14.5,'NORMAL PISTON K = ',I4//)

```

C DETERMINATION OF DISTRIBUTION OF SATURATION AND  
C OF OIL AND WATER RELATIVE PERMEABILITIES  
C

```

DEL2 = (SWIO - SWB)/DS
NDS = DS
NDSP1 = NDS + 1
NDSM1 = NDS - 1
SW(1) = SWIO
SW(NDSP1) = SWB
DO 110 N = 1,NDS
110 SW(N+1) = SW(N) - DEL2
DO 120 N = 1,NDSP1

```



## MAIN ••• (CONT'D)

```

CALL PERMA(N,SW,PO,PW,NDL)
120 F(N) = 1.0/(PO(N)/PW(N)*UWUO + 1.0)
    FP(1) = 0.0
    FP(NDSP1) = FPB
    DO 130 N = 2,NDS
130 FP(N) = (F(N-1) - F(N+1))/2.0/DEL2
    DEL3 = FPB/DS
    FPP(1) = 0.0
    FPP(NDSP1) = FPB
    DO 140 N = 1,NDSM1
140 FPP(N+1) = DEL3 + FPP(N)
    SWW(1) = SW(1)
    SWW(NDSP1) = SW(NDSP1)
    DO 150 N = 2,NDS
    DO 146 K = 2,NDSP1
        IF(FPP(N) - FP(K))148,144,146
144 SWW(N) = SW(K)
    GO TO 150
146 CONTINUE
148 SWW(N) = SW(K-1)+(SW(K)-SW(K-1))/(FP(K)-FP(K-1))
    1*(FPP(N)-FP(K-1))
150 CONTINUE
    DO 180 N = 1,NDSP1
        FP(N) = FPP(N)
        SW(N) = SWW(N)
        CALL PERMA(N,SW,PO,PW,NDL)
180 F(N) = 1.0/(PO(N)/PW(N)*UWUO + 1.0)
190 WRITE(6,1902)
1902 FORMAT('0',13X,'SW',17X,'PW',18X,'PO',18X,'F',16X,'FP'
    1//)
    WRITE(6,1904)(SW(N),PW(N),PO(N),F(N),FP(N),N = 1,NDSP1
    1,NSPCE)
1904 FORMAT(5F20.7)

```

C COMPUTATION OF WATER OIL RATIOS, OIL VOLUMES,  
C AND OTHER RELATED INTERMEDIATE DATA  
C

```

200 DO 210 N = 1,NDSP1
    RPW(N) = 1.0/PW(N)
    IF(N.EQ.1)GO TO 210
210 RPO(N) = 1.0/PO(N)
    RPO(1) = RPO(2)
    UNITS = FP(2) - FP(1)
    DO 220 N = 1,NDS
        PAO(N) = (RPO(N) + RPO(N+1))/2.0*UNITS
220 PAW(N) = (RPW(N) + RPW(N+1))/2.0*UNITS
    GO TO (326,327,325),KCASE
326 QT(1) = 0.10000
    NDL = 1

```



## MAIN ... (CONT'D)

```

GO TO 323
327 QT(1) = 0.0
NDL = 0
GO TO 323
325 QT(1) = 0.0
NDL = 1
323 DO 328 I = 1,IST
DO 328 J = 1,500
R(I,J) = 0.0
WOR(I,J) = 0.0
328 CONTINUE
DO 329 I = 1,500
SR(I) = 0.0
OILRCV(I) = 0.0
WORTIO(I) = 0.0
PVINJ(I) = 0.0
OILRAT(I) = 0.0
VO(I) = 0.0
WATRAT(I) = 0.0
QT(I) = QT(1)
329 CONTINUE
DO 221 I = 1,IST
SUMG = 0.0
DO 222 J = 1,NCELLS
SUMG = SUMG + G(I,J)
222 CONTINUE
R(I,1) = SUMG*UO/POIW
221 CONTINUE
DO 223 I = 1,IST
223 SR(1) = SR(1) + 1.0/R(I,1)
IF(KCASE.EQ.1)GO TO 60
GO TO 68
60 PRDRP(1) = QT(1)/(KABS*SR(1))
GO TO 65
68 PRDRP(1) = PRSS/KABS
QT(1) = PRDRP(1)*KABS*SR(1)
65 VINAL = 0.0
DO 224 I = 1,IST
Q(I,1) = (KABS*PRDRP(1))/R(I,1)
VCELL(I) = PALE(I)/NCELLS
224 VINAL = PALE(I) + VINAL
SAT = SWBR - SWC
WRITE(6,225)
225 FORMAT('1',2X,'TIME',3X,'OIL-RATE',3X,'WATER-RATE',
13X,'WATER-OIL-RATIO',3X,'OIL-RECOVERY',3X,'PORE-VOL
1-INJ',
13X,'PRESS-DROP',3X,'ITRS REQD',3X,'CELLS INVADED IN')
WRITE(6,226)
226 FORMAT('0',104X,'CHNL-1',1X,'CHNL-2',1X,'CHNL-3',1X
1,'CHNL-4')

```



## MAIN ... (CONT'D)

C CALCULATION OF PRODUCTION HISTORY IN THE PRIMARY PHASE  
C

```

COMTIM = 0.0
M = 0
ICOND = 1
330 M = M+1
      MMM = ICOND - 1
      JP = M
      NDIVS = M
      NEND = NDSP1
      CALL AVPERM(NDIVS,NEND,RPW,PAW,PABRW,FP,UNITS,IPAVPM)
      CALL AVPERM(NDIVS,NEND,RPO,PAO,PABRO,FP,UNITS,IPAVPM)
      TIME(M) = (VCELL(1)*SAT)/Q(1,M)
      MP1 = M + 1
      SUMG = 0.0
      DO 350 K = MP1,NCELLS
350 SUMG = SUMG + G(1,K)
      DO 360 JJ = 1,NDIVS
360 R(1,MP1) = R(1,MP1) + G(1,JJ)/(PABRW(JJ)/UW +
     1 PABRO(JJ)/UO)
      R(1,MP1) = R(1,MP1) + SUMG*UO/POIW
      CALL FLOCHK(M,JP,MMM,ICOND)
      VOO(M) = SAT*VCELL(1)*V
      OILRAT(M) = 0.0
      SR(MP1) = 0.0
      VO(M) = 0.0
      DO 370 J = 1,IST
      OILRAT(M) = OILRAT(M) + (Q(J,MP1) + Q(J,M))/2.0
      SR(MP1) = SR(MP1) + 2.0/(R(J,MP1) + R(J,M))
      IF(J.EQ.IST)GO TO 370
      VO(M) = VO(M) + (SAT*VCELL(J+1)*FRCELL(J+1))
370 CONTINUE
      VO(M) = VO(M) + VOO(M)
      IF(M-2)377,378,379
377 COMTIM = COMTIM + (TIME(M))/2.0
      OILRCV(M) = VO(M)/2.0/VINAL
      GO TO 371
378 COMTIM = COMTIM + (TIME(M-1) + TIME(M))/2.0
      OILRCV(M) = OILRCV(M-1) + (VO(M)/2.0)/VINAL
      GO TO 371
379 COMTIM = COMTIM + (TIME(M-1) + TIME(M))/2.0
      OILRCV(M) = OILRCV(M-1) + ((VO(M) - VO(M-1))/2.0)
     1/VINAL
371 PVINJ(M) = OILRCV(M)
      GO TO (376,342,340),KCASE
342 CALL FLORTE(COMTIM,MP1,QT,NDL)
      GO TO 376
340 QT(MP1) = OILRAT(M)

```



## MAIN ... (CONT'D)

376 PRDRP(MP1) = QT(MP1)\*1.0/(SR(MP1)\*KABS)  
 T(M) = COMTIM

C PRINT OUT OF RESULTS IN THE PRIMARY PHASE  
 C

```

  WRITE(6,400)COMTIM,OILRAT(M),WATTRAT(M),WORTIO(M),
  1OILRCV(M),PVINJ(M),PRDRP(MP1),KOUNT,NDIVS,(FRCELL(I),
  1I = 2,4)
400 FORMAT('0',1X,E10.3,1X,F7.5,2X,F7.5,6X,E10.3,3X,
  1E12.5,3X,E12.5,3X,E11.4,5X,I4,3X,I6,1X,F6.3,1X,
  1F6.3,1X,F6.3)
  WRITE(7,700)COMTIM,OILRCV(M),PVINJ(M),WORTIO(M)
  1,PRDRP(MP1)
700 FORMAT(E12.5,4X,E12.5,4X,E12.5,4X,E12.5,4X,E11.4)
  NN = NN + 1
  IF(NN.EQ.28)GO TO 30
  GO TO 35
30 WRITE(6,225)
  WRITE(6,226)
  WRITE(6,227)
227 FORMAT('0',55X,'PRIMARY - RECOVERY')
  NN = 0
35 IF(M.EQ.40)GO TO 401
  GO TO 330

```

C CALCULATION OF PRODUCTION HISTORY IN THE SUBORDINATE  
 C PHASE  
 C

```

401 IF(WORTIO(M).GE.12.0)GO TO 431
  M = M + 1
  MM = M
  JP = M
  MMM = ICOND - 1
  GO TO 461
431 ICOND = 0
  M = M + 1
  NSEND = KTP - 2
  MMM = ICOND - 1
  JK = IFIX((FLOAT(NEND)*FLOAT(NCELLS))/FLOAT(NSEND))
  JP = JK - M
  MM = M*ICOND + (M+JP - 1)*IABS(MMM)
461 NSEND = IFIX((FLOAT(NEND)*FLOAT(NCELLS))/FLOAT(MM))
  KTP = NSEND
  IF(NSEND.LE.2)GO TO 888
  NDIVS = NCELLS
  CALL AVPERM(NDIVS,NSEND,RPW,PAW,PABRW,FP,UNITS,IPAVPM)
  CALL AVPERM(NDIVS,NSEND,RPO,PAO,PABRO,FP,UNITS,IPAVPM)
  TIME(M) = (VCELL(1)*SAT*FLOAT(ICOND))/Q(1,M)+(VCELL(1)

```



## MAIN ... (CONT'D)

```

1*SAT* JP *
1FLOAT(IABS(MMM))/Q(1,M)
MP1 = M + 1
DO 402 JJ = 1,NDIVS
402 R(1,MP1) = R(1,MP1) + G(1,JJ)/(PABRW(JJ)/UW +
1 PABRO(JJ)/UC)
AVGSAT = SW(NSEND) + ((1.0 - F(NSEND))/FP(NSEND))
WOR(1,M) = PW(NSEND)/UWUO/PO(NSEND)
VOO(M) = (AVGSAT - SWBR)*PALE(1) + VOO(NCELLS)
CALL FLOCHK(M,JP,MMM,ICOND)
NNQ = 1
VO(M) = 0.0
WATRAT(M) = 0.0
OILRAT(M) = 0.0
DO 409 KL = 2,4
IF((FRCELL(KL) - NCELLS).GE.0.0)GO TO 411
GO TO 409
411 NNQ = KL
FRCELL(KL) = 40.0
409 CONTINUE
IF(NNQ.EQ.1)GO TO 417
DO 412 N = 2,NNQ
NSEND = IFIX((FLOAT(NEND)*FLOAT(NCELLS))/FCELL(N))
AVGSAT = SW(NSEND) + (1.0 - F(NSEND))/FP(NSEND)
WOR(N,M) = PW(NSEND)/UWUO/PO(NSEND)
VO(M) = VO(M) + (AVGSAT - SWBR)*PALE(N) + SAT*PALE(N)
412 CONTINUE
IF(NNQ.EQ.4)GO TO 450
417 NNQP1 = NNQ + 1
DO 418 K = NNQP1,IST
OILRAT(M) = OILRAT(M) + ((Q(K,MP1) + Q(K,M))/2.0)
VO(M) = VO(M) + (SAT*VCELL(K)*FCELL(K))
418 CONTINUE
450 VO(M) = VO(M) + VOO(M)
OILRCV(M) = OILRCV(M-1) + ((VO(M) - VO(M-2))/2.0)
1/VINAL
DO 421 KK = 1,NNQ
QAVG = (Q(KK,MP1) + Q(KK,M))/2.0
AVGWOR = (WOR(KK,M) + WOR(KK,M-1))/2.0
OILRAT(M) = OILRAT(M) + (QAVG/(1.0 + AVGWOR))
WATRAT(M) = WATRAT(M) + (QAVG*AVGWOR)/(1.0 + AVGWOR)
1 )
421 CONTINUE
COMTIM = COMTIM + (TIME(M) + TIME(M-1))/2.0
GO TO (422,440,445),KCASE
440 CALL FLORTE(COMTIM,MP1,QT,NDL)
GO TO 422
445 QT(MP1) = OILRAT(M) + WATRAT(M)
422 PVINJ(M) = PVINJ(M-1) + (((TIME(M)*WATRAT(M) +
1 TIME(M-1)*WATRAT(M-1))/2.0) + ((VO(M) - VO(M-2)))

```



## MAIN ... (CONT'D)

```

1/2.0))/VINAL
DO 423 KN = 1,IST
SR(MP1) = SR(MP1) + 2.0/(R(KN,MP1) + R(KN,M))
423 CONTINUE
PRDRP(MP1) = QT(MP1)*1.0/(SR(MP1)*KABS)
WORTIO(M) = WATRAT(M)/OILRAT(M)

C PRINT OUT OF RESULTS IN THE SUBORDINATE PHASE
C

495 WRITE(6,400)COMTIM,OILRAT(M),WATRAT(M),WORTIO(M),
1OILRCV(M),PVINJ(M),PRDRP(MP1),KOUNT,NDIVS,
1(FRCELL(I),I = 2,4)
      WRITE(7,700)COMTIM,OILRCV(M),PVINJ(M),WORTIO(M)
1,PRDRP(MP1)
ML = 0
T(M) = COMTIM
NN = NN + 1
IF(NN.EQ.28)GO TO 31
GO TO 37
31 WRITE(6,225)
      WRITE(6,226)
      WRITE(6,335)
335 FORMAT('0',53X,'SECONDARY - RECOVERY')
NN = 0
37 IF(PVINJ(M).GE.3.24)GO TO 991
      GO TO 401
991 GO TO (990,994,994),KCASE
994 KCASE = 1
      GO TO 326
888 WRITE(6,889)
889 FORMAT('0', 'MAX. WATER OIL RATIO UNREACHABLE')
990 STOP
END

```



## SUBROUTINE PERMA

SUBROUTINE PERMA(N,SW,PO,PW,NDL)

C THIS SUBROUTINE INTERPOLATES THE RELATIVE  
 C PERMEABILITIES ACCORDING TO THE SATURATION GRID  
 C

```

DIMENSION SW(510),PO(510),PW(510),YO(110),YW(110),
1X(110),D1(110),D2(110),D3(110),D11(110),
1D22(110),D33(110)
IF(NDL)100,100,200
100 NDL = 1
      READ(5,1001)JPOINT,ENDPO,ENDPW,ENDSW
1001 FORMAT(I3,3E12.5)
      READ(5,1002)(YO(J), J = 10,JPOINT)
      READ(5,1002)(YW(J), J = 10,JPOINT)
      READ(5,1002)( X(J), J = 10,JPOINT)
1002 FORMAT(6E12.5)
      JPNTM1 = JPOINT - 1
      JPNTM2 = JPOINT - 2
      JPNTM3 = JPOINT - 3
      DO 110 J = 10,JPNTM1
110 D1(J) = YO(J+1) - YO(J)
      DO 120 J = 10,JPNTM2
120 D2(J) = D1(J+1) - D1(J)
      DO 130 J = 10,JPNTM3
130 D3(J) = D2(J+1) - D2(J)
      DO 160 J = 10,JPNTM1
160 D11(J) = YW(J+1) - YW(J)
      DO 170 J = 10,JPNTM2
170 D22(J) = D11(J+1) - D11(J)
      DO 180 J = 10,JPNTM3
180 D33(J) = D22(J+1) - D22(J)
200 IF(ENDSW-SW(N))300,300,210
210 XX = SW(N)
      CALL STIRLG(JPOINT,YO,XX,X,D1,D2,D3,YY)
      PO(N) = YY
      CALL STIRLG(JPOINT,YW,XX,X,D11,D22,D33,YY)
      PW(N) = YY
      GO TO 500
300 SW(N) = ENDSW
      PO(N) = ENDPO
      PW(N) = ENDPW
500 RETURN
      END

```



## SUBROUTINE STIRLG

SUBROUTINE STIRLG(JPOINT,Y,XX,X,D1,D2,D3,YY)

C THIS SUBROUTINE DETERMINES WHICH OF THE FOLLOWING  
 C EQUATIONS SHOULD BE USED TO INTERPOLATE THE RELATIVE  
 C PERMEABILITIES FROM THE RELATIVE PERMEABILITY VERSUS  
 C SATURATION DATA

- C - NEWTONS FORWARD DIFFERENCE EQUATION
- C - STIRLING EQUATION
- C - NEWTONS BACKWARD DIFFERENCE EQUATION

```

DIMENSION Y(110),X(110),D1(110),D2(110),D3(110)
H = X(11) - X(10)
DO 70 J = 11,JPOINT
  IF(X(J) = XX)70,80,90
70 CONTINUE
  GO TO 90
80 YY = Y(J)
  GO TO 500
90 MM = J-1
  IF(MM = 12)100,200,300
100 U = (XX - X(MM))/H
  UPA = (U - 1.0)*U/2.0
  YY = Y(MM) + U*D1(MM)+UPA*D2(MM)+UPA*(U-2.0)*D3(MM)
  1/3.0
  GO TO 500
200 U = (XX-X(MM))/H
  YY = Y(MM)+(D1(MM-1)+D1(MM))*U/2.0+U**2*D2(MM-1)/2.0+U
  1*(U**2-1.0)
  1(D3(MM-2)+D3(MM-1))/12.0
  GO TO 500
300 IF(MM+3-JPOINT)200,200,400
400 U = (XX-X(MM+1))/H
  UPB = U*(U+1.0)/2.0
  YY = Y(MM+1)+U*D1(MM)+UPB*D2(MM-1)+UPB*(U+2.0)*D3(MM
  1-2)/3.0
500 RETURN
END

```



## SUBROUTINE AVPERM

SUBROUTINE AVPERM(NDIVS,NEND,RP,PA,PABR,FP,UNITS  
1,IPAVPM)

C THIS SUBROUTINE IS USED TO DETERMINE THE MEAN  
C PERMEABILITY TO OIL AND WATER WHEN THE NUMBER OF CELLS  
C INVADED IS A WHOLE NUMBER  
C

```
DIMENSION RP(510),PA(510),PABRL(510),FP(510),
1PAM(510),RPA(510),ALPA(510),PABR(510),RSWDIV(510),
1PBWN(510),SMUNIT(510)
1 DNDIVS = NDIVS
    WDIV = FP(NEND)/DNDIVS
    IF(2.0*UNITS - WDIV)300,300,200
```

C CALCULATION OF MEAN PERMEABILITIES WHEN NUMBER  
C OF DIVISIONS IS LOW - HIGH WATER OIL RATIO  
C

```
200 NDVSM1 = NDIVS - 1
    RSWDIV(1) = WDIV
    SMUNIT(1) = UNITS
    DO 205 N = 2,NEND
205 SMUNIT(N) = SMUNIT(N-1) + UNITS
    DO 210 N = 2,NDIVS
210 RSWDIV(N) = RSWDIV(N-1) + WDIV
    DO 260 K = 1,NDVSM1
    DO 240 NN = 1,NEND
        N = NN
        IF(SMUNIT(N) - RSWDIV(K))240,240,250
240 CONTINUE
250 IF(N-1)251,251,252
251 PBWN(K+1) = RP(1) - (RP(1)-RP(2))*RSWDIV(K)/UNITS
    GO TO 260
252 PBWN(K+1) = RP(N)-(RP(N)-RP(N+1))*(RSWDIV(K)-SMUNIT(N-1))/UNITS
    1CONTINUE
260 CONTINUE
    PBWN(1) = RP(1)
    PBWN(NDIVS+1) = RP(NEND + 1)
    DO 270 K = 1,NDIVS
270 PABRL(K) = (PBWN(K) + PBWN(K+1))/2.0
    GO TO 600
```

C CALCULATION OF MEAN PERMEABILITY WHEN NUMBER OF  
C DIVISIONS - IS HIGH  
C

```
300 IF(NDIVS - 1)310,310,340
310 J = 1
    NENDM1 = NEND - 1
```



## SUBROUTINE AVPERM... (CONT'D)

```

PAM(J) = 0.0
DO 320 K = 1,NENDM1
320 PAM(J) = PAM(J) + PA(K)
PAPRL(J) = PAM(J)/WDIV
GO TO 600
340 J = 1
SPWNDM = WDIV
DO 350 K = 1,NEND
IF(FP(K) = WDIV)350,360,360
350 CONTINUE
360 NT = K - 1
RPAPT = WDIV - FP(NT)
RPA(J) = (RP(NT)-(RPAPT/UNITS*(RP(NT)-RP(NT+1)))/2.0)
1*RPAPT
PAM(J) = 0.0
NTM1 = NT - 1
IF(NTM1 = 1)380,370,370
370 DO 375 K = 1,NTM1
375 PAM(J) = PAM(J) + PA(K)
380 PABRL(J) = (PAM(J) + RPA(J))/WDIV
400 DO 500 J = 2,NDIVS
PAM(J) = 0.0
DJ = J
RSPWM = SPWNDM
SPWNDM = WDIV*DJ
ALPAPT = FP(NT + 1) - RSPWM
ALPA(J) = (RP(NT+1)+(ALPAPT/UNITS*(RP(NT)-RP(NT+1)))
1/2.0)*ALPAPT
NTTP = NT
IF(J-NDIVS)430,420,420
420 RPA(J) = 0.0
NTM1 = NEND - 1
NTTP1 = NTTP + 1
GO TO 450
430 DO 440 N = NTTP,NEND
IF(FP(N)-SPWNDM)440,445,445
440 CONTINUE
445 NT = N - 1
RPAPT = SPWNDM - FP(NT)
NTTP1 = NTTP+1
NTM1 = NT - 1
RPA(J) = (RP(NT)-(RPAPT/UNITS)*(RP(NT)-RP(NT+1))/2.0)
1*RPAPT
450 IF(NT-NTTP = 1)460,490,470
460 IF(NEND-NT+1)490,490,470
470 DO 480 K = NTTP1,NTM1
480 PAM(J) = PAM(J) + PA(K)
490 PABRL(J) = (PAM(J)+ALPA(J)+RPA(J))/WDIV
500 CONTINUE
500 DO 610 N = 1,NDIVS

```



## SUBROUTINE AVPERM... (CONT'D)

```
610 PABR(N) = 1.0/PABRL(N)
    IF(1 - IPAVPM)620,620,630
620 WRITE(6,6200)NDIVS,(PABR(N),N = 1,NDIVS)
6200 FORMAT(//! MEAN PERMEABILITIES !,5X,! NO. CELLS
      1 INVADED !,I = !,I
      1/(5E15.5))
630 RETURN
END
```



## SUBROUTINE FLORTE

SUBROUTINE FLORTE(ELTIME,MP1,QT,NDL)

C THIS SUBROUTINE IS USED TO INTERPOLATE FOR INJECTION  
 C RATE FROM TI- ME VERSUS INJECTION RATE HISTORY AND IS  
 C USED IN A CASE WHEN BOTH INJECTION RATE AND PRESSURE  
 C DROP VARY

```

DIMENSION TIME(500),QT(500),X(110),Y(110),D1(110)
1,D2(110),D3(110)
  IF(NDL)100,100,190
100 NDL = 1
  READ(5,1001)JPOINT,ENDTIM,ENDQT
1001 FORMAT(I3,2E12.5)
  READ(5,1002)( X(J), J = 10,JPOINT)
  READ(5,1002)( Y(J), J = 10,JPOINT)
1002 FORMAT(6E12.5)
  JPNTM1 = JPOINT - 1
  JPNTM2 = JPOINT - 2
  JPNTM3 = JPOINT - 3
  DO 110 J = 10,JPNTM1
110 D1(J) = Y(J+1) - Y(J)
  DO 120 J = 10,JPNTM2
120 D2(J) = D1(J+1) - D1(J)
  DO 130 J = 10,JPNTM3
130 D3(J) = D2(J+1) - D2(J)
190 IF(ELTIME - X(10))5,5,200
  5 QT(MP1) = Y(10)
  GO TO 500
200 IF(ENDTIM - ELTIME)300,300,210
210 XX = ELTIME
  CALL STIRLG(JPOINT,Y ,XX,X,D1,D2,D3,YY)
  QT(MP1) = YY
  GO TO 500
300 QT(MP1) = ENDQT
500 RETURN
END

```



## SUBROUTINE FLOCHK

SUBROUTINE FLOCHK(M,JP,MMM,ICOND)

C THIS SUBROUTINE IS USED TO DETERMINE FLOW RATES IN  
 C VARIOUS CHANNELS IN A MANNER SO THAT CROSS-FLOW  
 C BETWEEN THE CHANNELS IS MINIMIZED

```

COMMON SW(510),SKW(510),PW(510),PO(510),RPW(510),
1RPO(510),PAW(510),PAO(510),PABRW(510),PABRO(510),
1F(510),FP(510),FPP(510),G(4,40),Q(4,500),TIME(500),
1R(4,500),FRCELL(4),VCELL(4),FCELL(4),QT(500),
1PRDRP(500)
COMMON SAT,NCELLS,UQ,UW,POIW,UNITS,NEND,IPAVPM,
1NDL,KOUNT,KCASE,COMTIM,MNQ
REAL KABS
KABS = 11.1
KOUNT = 1
III = 0
NSEND = NEND
MP1 = M + 1
DO 5 I = 1,4
5 Q(I,MP1) = Q(I,M)
IF(M-2)10,21,21
10 DO 15 I = 1,4
15 FCELL(I) = 0.0
21 NQ = 2
20 DO 25 L = NQ,4
  FRCELL(L) = FCELL(L)+(((Q(L,M)+Q(L,MP1))/2.0)*TIME(M))
  1/(VCELL(L)*
1SAT)
NSEND = NEND
IF(FRCELL(L).GE.40.0)GO TO 300
GO TO 301
300 NSEND = IFIX((FLOAT(NEND)*FLOAT(NCELLS))/FRCELL(L))
NDIVS = 40
GO TO 19
301 NDELL = IFIX(FRCELL(L))
RCELL = FRCELL(L) - FLOAT(NDELL)
DEV = FRCELL(L)/501.
IF(RCELL.LE.DEV)RCELL = 0.0
751 IF(FRCELL(L).LT.1.0)GO TO 17
  IF(RCELL.NE.0.0)GO TO 30
  NDIVS = NDELL
  NDLP1 = NDELL + 1
  GO TO 19
17 NDIVS = 1
19 CALL AVPERM(NDIVS,NSEND,RPW,PAW,PABRW,FP,UNITS,IPAVPM)
  CALL AVPERM(NDIVS,NSEND,RPO,PAO,PABRO,FP,UNITS,IPAVPM)
  IF(FRCELL(L).LT.1.0)GO TO 71
  IF(FRCELL(L).GE.40.0)GO TO 303

```



## SUBROUTINE FLOCHK... (CONT'D)

```

R(L,MP1) = 0.0
SUMG = 0.0
DO 100 K = NDLP1,NCELLS
100 SUMG = SUMG + G(L,K)
GO TO 302
303 SUMG = 0.0
R(L,MP1) = 0.0
302 DO 110 JJ = 1,NDIVS
110 R(L,MP1) = R(L,MP1) + G(L,JJ)/(PABRW(JJ)/UW +
1 PABRO(JJ)/UO)
R(L,MP1) = R(L,MP1) + SUMG * UO/POIW
GO TO 25
30 CALL AVPER2(NDELL,NEND,L,FRCELL,RPW,PAW,PABRW,FP,UNITS
1,IPAVPM)
CALL AVPER2(NDELL,NEND,L,FRCELL,RPO,PAO,PABRO,FP,UNITS
1,IPAVPM)
71 K2 = NDELL + 2
K1 = NDELL + 1
400 SUMG = 0.0
IF(K2.GT.40.0)GO TO 205
DO 200 I = K2,NCELLS
200 SUMG = SUMG + G(L,I)
205 SUMG = SUMG + G(L,K1)*(1.0 - RCELL)
410 R(L,MP1) = 0.0
DO 210 J = 1,K1
IF(J.EQ.K1)GO TO 215
R(L,MP1) = R(L,MP1) + G(L,J)/(PABRW(J)/UW + PABRO(J)
1/UO)
GO TO 210
215 R(L,MP1) = R(L,MP1)+G(L,J)*RCELL/(PABRW(J)/UW +
1 PABRO(J)/UO)
210 CONTINUE
R(L,MP1) = R(L,MP1) + SUMG*UO/POIW
25 CONTINUE
GO TO (2,2,3),KCASE
2 PROD = 0.0
DO 27 K = 1,4
QAVG = ( Q(K,M) + Q(K,MP1))/2.0
RAVG = ( R(K,MP1) + R(K,M))/2.0
PROD = PROD + QAVG*RAVG
IF((KOUNT.EQ.10).AND.(K.EQ.1))GO TO 700
27 CONTINUE
PROD = PROD/4.0
AVPROD = PROD
GO TO 725
700 PROD = (PROD + AVPROD)/2.0
KOUNT = 0
III = III + 1
725 DO 32 N = 1,4
QAVG = ( Q(N,M) + Q(N,MP1))/ 2.0

```



## SUBROUTINE FLOCHK... (CONT'D)

```

RAVG = ( R(N,M) + R(N,MP1) ) / 2.0
CHECK = ( QAVG * RAVG - PROD ) / PROD
IF( III.GE.1 ) GO TO 28
GO TO 29
28 WRITE(6,18) CHECK
18 FORMAT('0',4X,' CHECK = ',E13.6)
IPAVPM = 1
WRITE(6,45)(FRCELL(L),L = 2,4)
45 FORMAT('0',3(2X,E13.6))
777 FORMAT('0',1X,E13.6,4(1X,E13.6),4(1X,E13.6))
29 IF( ABS(CHECK).GE.1.E-04 ) GO TO 22
32 CONTINUE
GO TO 280
22 DO 33 I = 1,4
QAVG = ( Q(I,M) + Q(I,MP1) ) / 2.0
RAVG = ( R(I,M) + R(I,MP1) ) / 2.0
QAVG = QAVG - (( QAVG * RAVG - PROD ) / RAVG )
Q(I,MP1) = 2.0 * QAVG - Q(I,M)
33 CONTINUE
705 TIME(M) = ( VCELL(1) * SAT * FLOAT(ICOND) ) / ( ( Q(1,M) +
1Q(1,MP1) ) / 2.0 ) + ( VCELL(1) * SAT * JP * FLOAT(IABS(MMM
1)) ) / ( ( Q(1,MP1) + Q(1,M) ) / 2.0 )
IF( KCASE.EQ.1 ) GO TO 711
ELTIME = COMTIM + TIME(M)
IF( MNQ.GE.1 ) GO TO 710
IF( ELTIME.GT.0.13893E 04 ) GO TO 703
GO TO 710
703 MNQ = MNQ + 1
NDL = 0
710 CALL FLORTE(ELTIME,MP1,QT,NDL)
711 SUM = 0.0
DO 43 II = 1,4
43 SUM = SUM + Q(II,MP1)
IF( KCASE.EQ.1 ) GO TO 712
RERROR = (( SUM - QT(MP1) ) / QT(MP1))
IF( ABS(RERROR).LE.1.E-04 ) GO TO 707
712 DO 44 KK = 1,4
Q(KK,MP1) = ( Q(KK,MP1) * QT(MP1) ) / SUM
44 CONTINUE
IF( KCASE.EQ.1 ) GO TO 707
GO TO 705
707 KOUNT = KOUNT + 1
IF( KOUNT.EQ.80 ) GO TO 310
GO TO 20
280 KOUNT = KOUNT + III*10
GO TO 130
3 DO 120 K = 1,4
QAVG = ( Q(K,M) + Q(K,MP1) ) / 2.0
RAVG = ( R(K,M) + R(K,MP1) ) / 2.0

```



## SUBROUTINE FLOCHK... (CONT'D)

```
PRDCAL = (QAVG*RAVG)/KABS
PRCHK = (PRDCAL - PRDRP(1))/PRDRP(1)
IF(ABS(PRCHK).GE.1.E-05)GO TO 122
120 CONTINUE
GO TO 130
122 TIME(M) = (VCELL(1)*SAT*FLOAT(ICOND))
1/((Q(1,M)+Q(1,MP1))/2.0)+(VCELL(1)*SAT*JP*
1FLOAT(IABS(MMM)))/((Q(1,MP1) + Q(1,1))/2.0)
QAVG = (QAVG*PRDRP(1))/PRDCAL
Q(K,MP1) = (2.0*QAVG - Q(K,M))
KOUNT = KOUNT + 1
IF(KOUNT.EQ.80)GO TO 310
GO TO 20
130 DO 290 L = 1,4
290 FCELL(L) = FRCELL(L)
310 RETURN
END
```



## SUBROUTINE AVPER2

SUBROUTINE AVPER2(NDELL,NEND,L,FRCELL,RP,PA,PABR,FP  
1,UNITS,IPAVPM)

C THIS SUBROUTINE IS USED TO DETERMINE THE MEAN  
C PERMEABILITY TO OIL AND WATER IN A FRACTIONAL  
C CELL

```
DIMENSION FRCELL(4),RP(510),PA(510),PABRL(510),
1FP(510),PAM(510),RPA(510),ALPA(510),PABR(510),
1RSWDIV(51),PBWN(51),SMUNIT(510),
1RSWDIV(51),PBWN(51),SMUNIT(510)
IEND = IFIX(FLOAT(NEND)*((FRCELL(L)-FLOAT(NDELL))
1/FRCELL(L)))
NDEND = NEND - IFND
WDIV = FP(NDEND)/FLOAT(NDELL)
NPDEL1 = NDELL + 1
IF(2.0*UNITS - WDIV)300,300,200
```

C CALCULATION OF MEAN PERMEABILITIES WHEN NUMBER  
C OF DIVISION-  
C S IS LOW - HIGH WATER OIL RATIO

```
200 NDLLM1 = NDELL - 1
      RSWDIV(1) = WDIV
      SMUNIT(1) = UNITS
      DO 205 N = 2,NDEND
205 SMUNIT(N) = SMUNIT(N-1) + UNITS
      DO 210 N = 2,NDELL
210 RSWDIV(N) = RSWDIV(N-1) + WDIV
      DO 260 K = 1,NDLLM1
      DO 240 NN = 1,NDEND
      N = NN
      IF(SMUNIT(N) - RSWDIV(K))240,240,250
240 CONTINUE
250 IF(N-1)251,251,252
251 PBWN(K+1) = RP(1) - (RP(1)-RP(2))*RSWDIV(K)/UNITS
      GO TO 260
252 PBWN(K+1) = RP(N)-(RP(N)-RP(N+1))*(RSWDIV(K)-SMUNIT(N
      1-1))/UNITS
260 CONTINUE
      PBWN(1) = RP(1)
      PBWN(NDELL+1) = RP(NDEND+1)
      DO 270 K = 1,NDELL
270 PABRL(K) = (PBWN(K) + PBWN(K+1))/2.0
275 ALOK = 0.0
      NENDM1 = NEND - 1
      DO 280 K = NDEND,NENDM1
280 ALOK = ALOK + PA(K)
      PABRL(NPDEL1) = ALOK/(FP(NEND) - FP(NDEND)))
```



## SUBROUTINE AVPER2... (CONT'D)

GO TO 600

C CALCULATION OF MEAN PERMEABILITY WHEN NUMBER OF  
C DIVISIONS -  
C IS HIGH

```

300 IF(NDELL - 1)310,310,340
310 J = 1
    NDNDM1 = NDEND - 1
    PAM(J) = 0.0
    DO 320 K = 1,NDNDM1
320 PAM(J) = PAM(J) + PA(K)
    PABRL(J) = PAM(J)/WDIV
    GO TO 275
340 J = 1
    SPWNMD = WDIV
    DO 350 K = 1,NDEND
    IF(FP(K) - WDIV)350,360,360
350 CONTINUE
360 NT = K - 1
    RPAPT = WDIV - FP(NT)
    RPA(J) = (RP(NT)-(RPAPT/UNITS*(RP(NT)-RP(NT+1)))/2.0)
1*RPAPT
    PAM(J) = 0.0
    NTM1 = NT - 1
    IF(NTM1 - 1)380,370,370
370 DO 375 K = 1,NTM1
375 PAM(J) = PAM(J) + PA(K)
380 PABRL(J) = (PAM(J) + RPA(J))/WDIV
400 DO 500 J = 2,NDELL
    PAM(J) = 0.0
    DJ = J
    RSPWM = SPWNMD
    SPWNMD = WDIV*DJ
    ALPAPT = FP(NT + 1) - RSPWM
    ALPA(J) = (RP(NT+1)+(ALPAPT/UNITS*(RP(NT)-RP(NT+1)))1/2.0)*ALPAPT
    IF(J-NDELL)430,420,420
420 RPA(J) = 0.0
    NTM1 = NDEND - 1
    NTTP1 = NTTP + 1
    GO TO 450
430 DO 440 N = NTTP,NDEND
    IF(FP(N)-SPWNMD)440,445,445
440 CONTINUE
445 NT = N - 1
    RPAPT = SPWNMD - FP(NT)
    NTTP1 = NTTP+1
    NTM1 = NT - 1
    RPA(J) = (RP(NT)-(RPAPT/UNITS)*(RP(NT)-RP(NT+1))/2.0)

```



## SUBROUTINE AVPER2... (CONT'D)

```
1*RPAPT
450 IF(INT-NTTP - 1)450,490,470
460 IF(NDEND-NT+1)490,490,470
470 DO 480 K = NTTP1,NTM1
480 PAM(J) = PAM(J) + PA(K)
490 PABRL(J) = (PAM(J)+ALPA(J)+RPA(J))/WDIV
500 CONTINUE
      GO TO 275
500 DO 610 N = 1,NPDEL1
510 PAPR(N) = 1.C/PABRL(N)
      IF(1.0 - IPAVER)620,620,630
520 WRITE(6,6200)NDELL,(PABR(N),N = 1,NDELL)
5200 FORMAT(//'MEAN PERMEABILITIES',5X,'COMP. CELLS
      1INV. = ',I6,//(5E15.5))
      WRITE(6,6300)PABR(NPDEL1)
5300 FORMAT(// ' MEAN PERMEABILITY IN FRACTIONAL CELL = '
      1,E15.5)
      WRITE(6,6400)L
5400 FORMAT('0',4X,' CHANNEL NUMBER = ',I4)
530 RETURN
      END
```



## SUBROUTINE CAPLOT

SUBROUTINE CAPLOT(X3,Y3,X4,Y4,X5,Y5,X6,Y6,X7,Y7,KCPD,KCFR)

C THIS SUBROUTINE IS USED TO PLOT THE RESULTS  
 C

```

DIMENSION X(32),Y(32),X1(32),Y1(32),X2(32),Y2(32),
1X3(200),Y3(200),X4(200),Y4(200),X5(32),Y5(32),
1X6(200),Y6(200),X7(200),Y7(200),WORK(1024)
CALL PLOTS(WORK(1),4096)
READ(5,10)(X1(I),I = 1,30)
READ(5,10)(Y1(I),I = 1,30)
READ(5,10)(X2(I),I = 1,30)
READ(5,10)(Y2(I),I = 1,30)
READ(5,10)(X5(I),I = 1,30)
READ(5,10)(Y5(I),I = 1,30)
10 FORMAT(6E12.5)
DO 20 J = 1,30
X5(J) = X5(J)/10.0
20 CONTINUE
CALL PLOT( 0.0, 0.0, 3)
CALL PLOT( 0.0,12.0, 2)
CALL PLOT(18.0,12.0, 2)
CALL PLOT(18.0, 0.0, 2)
CALL PLOT(0.0,0.0,2)
CALL PLOT(1.0,1.0,-3)
CALL SCALE(X1,16.0, 30,1,20.0)
CALL SCALE(Y1,10.0, 30,1,20.0)
CALL SCALE(X2,16.0, 30,1,20.0)
CALL SCALE(Y2,10.0, 30,1,20.0)
CALL SCALE(X3,16.0,KCPD,1,20.0)
CALL SCALE(Y3,10.0,KCPD,1,20.0)
CALL SCALE(X4,16.0,KCFR,1,20.0)
CALL SCALE(Y4,10.0,KCFR,1,20.0)
XMIN = 0.0
XDELTA = 0.25
YMIN = 0.0
YDELTA = 0.08
X1(31) = XMIN
X1(32) = XDELTA
Y1(31) = YMIN
Y1(32) = YDELTA
CALL AXIS(0.0,0.0,'PORE VOLUMES WATER INJECTED',
1-27,16.0,0.0,X1(31),X1(32),20.0)
CALL AXIS(0.0,0.0,'PORE VOLUMES OIL PRODUCED',
125,10.0,90.0,Y1(31),Y1(32),20.0)
X2(31) = XMIN
X2(32) = XDELTA
X3(KCPD+1) = XMIN
X3(KCPD+2) = XDELTA
X4(KCFR+1) = XMIN

```



## SUBROUTINE CAPLOT... (CONT'D)

```

X4(KCFR+2) = XDELTA
Y2(31) = YMIN
Y2(32) = YDELTA
Y3(KCPD+1) = YMIN
Y3(KCPD+2) = YDELTA
Y4(KCFR+1) = YMIN
Y4(KCFR+2) = YDELTA
CALL LINE(X1,Y1,30,1,1,5)
CALL LINE(X2,Y2,30,1,1,2)
CALL LINE(X3,Y3,KCPD,1,1,11)
CALL SYMBOL(11.0,3.0,0.15,'LEGEND',0.0,6)
CALL SYMBOL(10.0,2.0,0.10,5,0.0,-1)
CALL SYMBOL(10.0,1.7,0.10,2,0.0,-1)
CALL SYMBOL(10.0,1.4,0.10,11,0.0,-1)
CALL SYMBOL(10.0,1.1,0.10,3,0.0,-1)
CALL SYMBOL(10.5,2.0,0.10,'DOUGLAS AND PEACEMANS DATA'
1,0.0,26)
CALL SYMBOL(10.5,1.7,0.10,'HIGGINS AND LEIGHTONS
1 RESULTS',0.0,29)
CALL SYMBOL(10.5,1.4,0.10,'CONST.PRESS DROP-
1PROPOSED METHOD',0.0,32)
CALL SYMBOL(10.5,1.1,0.10,'CONST.FLOW RATE-
1PROPOSED METHOD',0.0,31)
CALL SYMBOL(4.0,10.2,0.20,'PLOT OF OIL RECV
1ERY VS PORE-VOL-INJECTED',0.0,41)
WRITE(6,50)
50 FORMAT('0',10X,'AT THE MIDDLE OF THE SUBROUTINE')
CALL PLOT(23.0,-1.0,-3)
CALL PLOT( 0.0, 0.0, 3)
CALL PLOT( 0.0,12.0, 2)
CALL PLOT(18.0,12.0, 2)
CALL PLOT(18.0, 0.0, 2)
CALL PLOT(0.0,0.0,2)
CALL PLOT(1.0,1.0,-3)
CALL SCALE(X5,16.0, 30,1,20.0)
CALL SCALE(Y5,10.0, 30,1,20.0)
CALL SCALE(X6,16.0,KCFR,1,20.0)
CALL SCALE(Y6,10.0,KCFR,1,20.0)
CALL SCALE(X7,16.0,KCPD,1,20.0)
CALL SCALE(Y7,10.0,KCPD,1,20.0)
XMIN = 0.0
YMIN = 0.0
XDELTA = 70.0
YDELTA = 5.0
Y5(31) = YMIN
Y5(32) = YDELTA
X5(31) = XMIN
X5(32) = XDELTA
CALL AXIS(0.0,0.0,'TIME IN SECS (*10**X)',-21,
116.0,0.0,X5(31),X5(32),20.0)

```



## SUBROUTINE CAPLOT... (CONT'D)

```
CALL AXIS(0.0,0.0,'WATER-OIL-RATIO',15,10.0,90.0,  
1Y5(31),Y5(32),20.0)  
X6(KCFR+1) = XMIN  
X6(KCFR+2) = XDELTA  
X7(KCPD+1) = XMIN  
X7(KCPD+2) = XDELTA  
Y6(KCFR+1) = YMIN  
Y6(KCFR+2) = YDELTA  
Y7(KCPD+1) = YMIN  
Y7(KCPD+2) = YDELTA  
CALL LINE(X5,Y5,30,1,1,2)  
CALL LINE(X6,Y6,KCFR,1,1,11)  
CALL LINE(X7,Y7,KCPD,1,1,3)  
CALL SYMBOL(12.0,3.0,0.15,'LEGEND',0.0,6)  
CALL SYMBOL(11.0,2.0,0.10,2,0.0,-1)  
CALL SYMBOL(11.0,1.6,0.10,11,0.0,-1)  
CALL SYMBOL(11.0,1.2,0.10,3,0.0,-1)  
CALL SYMBOL(11.5,2.0,0.10,'HIGGINS AND LEIGHTONS  
RESULTS ,X = 1',0.0,36)  
CALL SYMBOL(11.5,1.6,0.10,'CONST.FLOW RATE-PROPOS  
1ED METHOD ,X = 2',0.0,38)  
CALL SYMBOL(11.5,1.2,0.10,'CONST.PRSS DROP-PROPOS  
1ED METHOD ,X = 1',0.0,38)  
CALL SYMBOL(5.0,10.5,0.20,'PLOT OF TIME VS WATER  
1-OIL-RATIO',0.0,31)  
CALL PLOT(24.0,-1.0,-3)  
CALL PLOT(0.0,0.0,999)  
RETURN  
END
```



APPENDIX - C

- Laboratory data on a five spot pattern reported by Douglas, Peaceman, and Rachford.
- Calculated results using Higgins-Leighton method.
- Calculated results using the modified scheme.







C - 2

HIGGINS - LEIGHTON RESULTS

TIME	OIL RECOVERY	PORE VOL. INJ.	WATER OIL RATIO
0.69466E 02	0.15212E-01	0.15212E-01	0.00000E 00
0.13893E 03	0.32500E-01	0.32500E-01	0.00000E 00
0.20840E 03	0.51195E-01	0.51195E-01	0.00000E 00
0.27787E 03	0.70662E-01	0.70662E-01	0.00000E 00
0.34733E 03	0.90615E-01	0.90615E-01	0.00000E 00
0.41680E 03	0.11095E 00	0.11095E 00	0.00000E 00
0.48627E 03	0.13160E 00	0.13160E 00	0.00000E 00
0.55573E 03	0.15253E 00	0.15253E 00	0.00000E 00
0.62520E 03	0.17369E 00	0.17369E 00	0.00000E 00
0.69466E 03	0.19506E 00	0.19506E 00	0.00000E 00
0.76413E 03	0.21664E 00	0.21664E 00	0.00000E 00
0.83360E 03	0.23839E 00	0.23839E 00	0.00000E 00
0.90306E 03	0.26032E 00	0.26032E 00	0.00000E 00
0.97253E 03	0.28243E 00	0.28243E 00	0.00000E 00
0.10420E 04	0.30472E 00	0.30472E 00	0.00000E 00
0.11115E 04	0.32720E 00	0.32720E 00	0.00000E 00
0.11809E 04	0.34991E 00	0.34991E 00	0.00000E 00
0.12504E 04	0.37289E 00	0.37289E 00	0.00000E 00
0.13199E 04	0.39648E 00	0.39722E 00	0.38571E 00
0.13893E 04	0.41508E 00	0.42481E 00	0.50020E 00
0.17367E 04	0.49048E 00	0.58995E 00	0.17101E 01
0.20840E 04	0.54310E 00	0.79620E 00	0.45229E 01
0.24313E 04	0.58170E 00	0.10349E 01	0.58487E 01
0.27787E 04	0.61655E 00	0.13004E 01	0.11991E 02
0.31260E 04	0.63663E 00	0.15975E 01	0.15720E 02
0.34733E 04	0.65342E 00	0.19167E 01	0.20369E 02
0.38207E 04	0.66754E 00	0.22552E 01	0.25644E 02
0.41680E 04	0.67954E 00	0.26104E 01	0.31202E 02
0.45153E 04	0.68997E 00	0.29794E 01	0.37119E 02
0.48627E 04	0.69919E 00	0.33595E 01	0.43544E 02



TIME	OIL RECOVERY	CONSTANT	PRESSURE	DROP	PROPOSED	METHOD
		PORE VOL.	INJ.	W.O.R.	PRESS.	DROP
0.27553E 02	0.47971E-02	0.47971E-02	0.00	0.900E 01		
0.78238E 02	0.14307E-01	0.14307E-01	0.00	0.900E 01		
0.12299E 03	0.23804E-01	0.23804E-01	0.00	0.900E 01		
0.16549E 03	0.33401E-01	0.33401E-01	0.00	0.900E 01		
0.20675E 03	0.43040E-01	0.43040E-01	0.00	0.900E 01		
0.24714E 03	0.52697E-01	0.52697E-01	0.00	0.900E 01		
0.28689E 03	0.62362E-01	0.62362E-01	0.00	0.900E 01		
0.32611E 03	0.72034E-01	0.72034E-01	0.00	0.900E 01		
0.36491E 03	0.81713E-01	0.81713E-01	0.00	0.900E 01		
0.40333E 03	0.91396E-01	0.91396E-01	0.00	0.900E 01		
0.44142E 03	0.10108E 00	0.10108E 00	0.00	0.900E 01		
0.47921E 03	0.11077E 00	0.11077E 00	0.00	0.900E 01		
0.51673E 03	0.12045E 00	0.12045E 00	0.00	0.900E 01		
0.55399E 03	0.13014E 00	0.13014E 00	0.00	0.900E 01		
0.59102E 03	0.13982E 00	0.13982E 00	0.00	0.900E 01		
0.62782E 03	0.14950E 00	0.14950E 00	0.00	0.900E 01		
0.66443E 03	0.15918E 00	0.15918E 00	0.00	0.900E 01		
0.70083E 03	0.16886E 00	0.16886E 00	0.00	0.900E 01		
0.73705E 03	0.17853E 00	0.17853E 00	0.00	0.900E 01		
0.77309E 03	0.18819E 00	0.18819E 00	0.00	0.900E 01		
0.80895E 03	0.19786E 00	0.19786E 00	0.00	0.900E 01		
0.84465E 03	0.20751E 00	0.20751E 00	0.00	0.900E 01		
0.88018E 03	0.21716E 00	0.21716E 00	0.00	0.900E 01		
0.91554E 03	0.22680E 00	0.22680E 00	0.00	0.900E 01		
0.95073E 03	0.23644E 00	0.23644E 00	0.00	0.900E 01		
0.98576E 03	0.24606E 00	0.24606E 00	0.00	0.900E 01		
0.10206E 04	0.25567E 00	0.25567E 00	0.00	0.900E 01		
0.10553E 04	0.26527E 00	0.26527E 00	0.00	0.900E 01		
0.10898E 04	0.27486E 00	0.27486E 00	0.00	0.900E 01		
0.11241E 04	0.28443E 00	0.28443E 00	0.00	0.900E 01		
0.11583E 04	0.29398E 00	0.29398E 00	0.00	0.900E 01		
0.11922E 04	0.30351E 00	0.30351E 00	0.00	0.900E 01		
0.12259E 04	0.31302E 00	0.31302E 00	0.00	0.900E 01		
0.12594E 04	0.32251E 00	0.32251E 00	0.00	0.900E 01		
0.12926E 04	0.33196E 00	0.33196E 00	0.00	0.900E 01		
0.13255E 04	0.34137E 00	0.34137E 00	0.00	0.900E 01		
0.13581E 04	0.35073E 00	0.35073E 00	0.00	0.900E 01		
0.13901E 04	0.36002E 00	0.36002E 00	0.00	0.900E 01		
0.14215E 04	0.36920E 00	0.36920E 00	0.00	0.900E 01		
0.14496E 04	0.37774E 00	0.37774E 00	0.00	0.900E 01		

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TIME	OIL RECOVERY	CONSTANT	PRESSURE	DROP	PROPOSED	METHOD
			PORE VOL.	INJ.	w.o.R.	PRESS.
0.14721E 04	0.38406E 00	0.38499E 00	0.36	0.900E 01		
0.14917E 04	0.38867E 00	0.39165E 00	0.49	0.900E 01		
0.15111E 04	0.39320E 00	0.39844E 00	0.50	0.900E 01		
0.15302E 04	0.39770E 00	0.40523E 00	0.51	0.900E 01		
0.15491E 04	0.40216E 00	0.41199E 00	0.52	0.900E 01		
0.15677E 04	0.40658E 00	0.41873E 00	0.52	0.900E 01		
0.15862E 04	0.41097E 00	0.42543E 00	0.53	0.900E 01		
0.16045E 04	0.41532E 00	0.43212E 00	0.53	0.900E 01		
0.16226E 04	0.41973E 00	0.43889E 00	0.53	0.900E 01		
0.16405E 04	0.42422E 00	0.44644E 00	1.10	0.900E 01		
0.16582E 04	0.42805E 00	0.45435E 00	1.38	0.900E 01		
0.16757E 04	0.43117E 00	0.46183E 00	1.41	0.900E 01		
0.16931E 04	0.43422E 00	0.46928E 00	1.43	0.900E 01		
0.17103E 04	0.43728E 00	0.47675E 00	1.46	0.900E 01		
0.17274E 04	0.44030E 00	0.48422E 00	1.48	0.900E 01		
0.17443E 04	0.44330E 00	0.49167E 00	1.50	0.900E 01		
0.17611E 04	0.44626E 00	0.49911E 00	1.53	0.900E 01		
0.17777E 04	0.44919E 00	0.50653E 00	1.55	0.900E 01		
0.17942E 04	0.45209E 00	0.51394E 00	1.57	0.900E 01		
0.18105E 04	0.45491E 00	0.52130E 00	1.60	0.900E 01		
0.18267E 04	0.45774E 00	0.52868E 00	1.62	0.900E 01		
0.18428E 04	0.46053E 00	0.53604E 00	1.64	0.900E 01		
0.18587E 04	0.46329E 00	0.54338E 00	1.66	0.900E 01		
0.18746E 04	0.46604E 00	0.55074E 00	1.69	0.900E 01		
0.18903E 04	0.46877E 00	0.55808E 00	1.71	0.900E 01		
0.19059E 04	0.47145E 00	0.56540E 00	1.73	0.900E 01		
0.19213E 04	0.47410E 00	0.57269E 00	1.75	0.900E 01		
0.19367E 04	0.47678E 00	0.58004E 00	1.77	0.900E 01		
0.19520E 04	0.47944E 00	0.58736E 00	1.79	0.900E 01		
0.19671E 04	0.48201E 00	0.59462E 00	1.80	0.900E 01		
0.19822E 04	0.48459E 00	0.60190E 00	1.82	0.900E 01		
0.19971E 04	0.48717E 00	0.60920E 00	1.84	0.900E 01		
0.20120E 04	0.48976E 00	0.61652E 00	1.85	0.900E 01		
0.20267E 04	0.49232E 00	0.62381E 00	1.86	0.900E 01		
0.20414E 04	0.49486E 00	0.63110E 00	1.86	0.900E 01		
0.20560E 04	0.49747E 00	0.63847E 00	1.79	0.900E 01		
0.20705E 04	0.50023E 00	0.64600E 00	1.64	0.900E 01		
0.20848E 04	0.50270E 00	0.65385E 00	3.11	0.900E 01		
0.20991E 04	0.50455E 00	0.66184E 00	3.81	0.900E 01		
0.21134E 04	0.50618E 00	0.66977E 00	3.86	0.900E 01		

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TIME	OIL RECOVERY	CONSTANT	PRESSURE	DROP	PROPOSED	METHOD
		PORE VOL.	INJ.	W.O.R.	PRESS.	DROP
0.21275E 04	0.50777E 00	0.67767E 00	3.91	0.900E 01		
0.21416E 04	0.50933E 00	0.68556E 00	3.96	0.900E 01		
0.21555E 04	0.51094E 00	0.69352E 00	4.01	0.900E 01		
0.21695E 04	0.51255E 00	0.70149E 00	4.06	0.900E 01		
0.21833E 04	0.51413E 00	0.70945E 00	4.11	0.900E 01		
0.21971E 04	0.51564E 00	0.71735E 00	4.16	0.900E 01		
0.22108E 04	0.51715E 00	0.72527E 00	4.21	0.900E 01		
0.22244E 04	0.51872E 00	0.73326E 00	4.26	0.900E 01		
0.22379E 04	0.52020E 00	0.74117E 00	4.31	0.900E 01		
0.22514E 04	0.52168E 00	0.74910E 00	4.36	0.900E 01		
0.22649E 04	0.52317E 00	0.75705E 00	4.41	0.900E 01		
0.22782E 04	0.52463E 00	0.76499E 00	4.46	0.900E 01		
0.22915E 04	0.52610E 00	0.77294E 00	4.51	0.900E 01		
0.23048E 04	0.52752E 00	0.78086E 00	4.56	0.900E 01		
0.23180E 04	0.52900E 00	0.78886E 00	4.62	0.900E 01		
0.23311E 04	0.53048E 00	0.79688E 00	4.67	0.900E 01		
0.23442E 04	0.53187E 00	0.80482E 00	4.71	0.900E 01		
0.23572E 04	0.53322E 00	0.81274E 00	4.76	0.900E 01		
0.23702E 04	0.53458E 00	0.82068E 00	4.80	0.900E 01		
0.23832E 04	0.53594E 00	0.82863E 00	4.85	0.900E 01		
0.23961E 04	0.53731E 00	0.83661E 00	4.90	0.900E 01		
0.24089E 04	0.53868E 00	0.84460E 00	4.95	0.900E 01		
0.24217E 04	0.54002E 00	0.85256E 00	4.99	0.900E 01		
0.24344E 04	0.54136E 00	0.86054E 00	5.04	0.900E 01		
0.24471E 04	0.54265E 00	0.86847E 00	5.09	0.900E 01		
0.24597E 04	0.54394E 00	0.87643E 00	5.13	0.900E 01		
0.24723E 04	0.54523E 00	0.88439E 00	5.18	0.900E 01		
0.24849E 04	0.54654E 00	0.89238E 00	5.23	0.900E 01		
0.24974E 04	0.54785E 00	0.90039E 00	5.28	0.900E 01		
0.25098E 04	0.54905E 00	0.90830E 00	5.32	0.900E 01		
0.25222E 04	0.55031E 00	0.91627E 00	5.36	0.900E 01		
0.25346E 04	0.55163E 00	0.92431E 00	5.41	0.900E 01		
0.25469E 04	0.55285E 00	0.93226E 00	5.46	0.900E 01		
0.25591E 04	0.55407E 00	0.94023E 00	5.50	0.900E 01		
0.25714E 04	0.55530E 00	0.94821E 00	5.55	0.900E 01		
0.25836E 04	0.55653E 00	0.95620E 00	5.59	0.900E 01		
0.25957E 04	0.55777E 00	0.96421E 00	5.64	0.900E 01		
0.26078E 04	0.55900E 00	0.97224E 00	5.69	0.900E 01		
0.26199E 04	0.56020E 00	0.98022E 00	5.73	0.900E 01		
0.26319E 04	0.56133E 00	0.98815E 00	5.77	0.900E 01		

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TIME	OIL RECOVERY	CONSTANT	PRESSURE	DROP	PROPOSED	METHOD
			PORE VOL.	INJ.	W.O.R.	PRESS.
0.26439E 04	0.56252E 00	0.99615E 00	5.81	0.900E 01		
0.26559E 04	0.56372E 00	0.10042E 01	5.85	0.900E 01		
0.26678E 04	0.56493E 00	0.10122E 01	5.89	0.900E 01		
0.26796E 04	0.56608E 00	0.10202E 01	5.94	0.900E 01		
0.26915E 04	0.56717E 00	0.10281E 01	5.97	0.900E 01		
0.27032E 04	0.56832E 00	0.10361E 01	6.02	0.900E 01		
0.27150E 04	0.56948E 00	0.10441E 01	6.07	0.900E 01		
0.27267E 04	0.57065E 00	0.10521E 01	6.12	0.900E 01		
0.27383E 04	0.57174E 00	0.10600E 01	6.16	0.900E 01		
0.27499E 04	0.57279E 00	0.10680E 01	6.19	0.900E 01		
0.27615E 04	0.57397E 00	0.10760E 01	6.24	0.900E 01		
0.27731E 04	0.57515E 00	0.10841E 01	6.28	0.900E 01		
0.27846E 04	0.57620E 00	0.10920E 01	6.32	0.900E 01		
0.27960E 04	0.57720E 00	0.10999E 01	6.35	0.900E 01		
0.28075E 04	0.57831E 00	0.11079E 01	6.39	0.900E 01		
0.28189E 04	0.57943E 00	0.11159E 01	6.43	0.900E 01		
0.28303E 04	0.58051E 00	0.11239E 01	6.46	0.900E 01		
0.28416E 04	0.58158E 00	0.11319E 01	6.50	0.900E 01		
0.28529E 04	0.58259E 00	0.11398E 01	6.53	0.900E 01		
0.28642E 04	0.58367E 00	0.11478E 01	6.57	0.900E 01		
0.28755E 04	0.58475E 00	0.11558E 01	6.60	0.900E 01		
0.28867E 04	0.58577E 00	0.11638E 01	6.63	0.900E 01		
0.28979E 04	0.58679E 00	0.11717E 01	6.66	0.900E 01		
0.29091E 04	0.58788E 00	0.11798E 01	6.70	0.900E 01		
0.29202E 04	0.58898E 00	0.11878E 01	6.73	0.900E 01		
0.29313E 04	0.59001E 00	0.11958E 01	6.76	0.900E 01		
0.29424E 04	0.59105E 00	0.12038E 01	6.78	0.900E 01		
0.29534E 04	0.59203E 00	0.12118E 01	6.81	0.900E 01		
0.29644E 04	0.59301E 00	0.12198E 01	6.83	0.900E 01		
0.29754E 04	0.59413E 00	0.12279E 01	6.86	0.900E 01		
0.29864E 04	0.59527E 00	0.12360E 01	6.74	0.900E 01		
0.29973E 04	0.59631E 00	0.12440E 01	6.47	0.900E 01		
0.30082E 04	0.59740E 00	0.12521E 01	6.20	0.900E 01		
0.30190E 04	0.59854E 00	0.12603E 01	5.86	0.900E 01		
0.30299E 04	0.59962E 00	0.12686E 01	9.78	0.900E 01		
0.30407E 04	0.60044E 00	0.12770E 01	11.60	0.900E 01		
0.30515E 04	0.60113E 00	0.12853E 01	11.72	0.900E 01		
0.30622E 04	0.60182E 00	0.12937E 01	11.83	0.900E 01		
0.30730E 04	0.60240E 00	0.13020E 01	11.92	0.900E 01		
0.30837E 04	0.60305E 00	0.13103E 01	12.01	0.900E 01		

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TIME	CONSTANT OIL RECOVERY	PRESSURE PORE VOL.	DROP INJ.	PROPOSED W.O.R.	METHOD PRESS. DROP
0.30944E 04	0.60372E 00	0.13186E 01	12.12	0.900E 01	
0.31104E 04	0.60470E 00	0.13312E 01	12.26	0.900E 01	
0.31422E 04	0.60659E 00	0.13562E 01	12.55	0.900E 01	
0.31897E 04	0.60909E 00	0.13936E 01	12.94	0.900E 01	
0.33207E 04	0.61554E 00	0.14983E 01	13.99	0.900E 01	
0.32475E 04	0.61194E 00	0.14395E 01	13.40	0.900E 01	
0.34142E 04	0.61991E 00	0.15745E 01	14.75	0.900E 01	
0.35277E 04	0.62495E 00	0.16685E 01	15.68	0.900E 01	
0.36612E 04	0.63052E 00	0.17810E 01	16.80	0.900E 01	
0.38094E 04	0.63633E 00	0.19085E 01	18.15	0.900E 01	
0.39770E 04	0.64252E 00	0.20560E 01	19.79	0.900E 01	
0.41688E 04	0.64902E 00	0.22287E 01	21.70	0.900E 01	
0.43844E 04	0.65581E 00	0.24277E 01	23.97	0.900E 01	
0.46284E 04	0.66297E 00	0.26588E 01	26.68	0.900E 01	
0.49004E 04	0.67009E 00	0.29229E 01	29.69	0.900E 01	
0.51999E 04	0.67709E 00	0.32209E 01	33.12	0.900E 01	
0.55314E 04	0.68412E 00	0.35590E 01	37.08	0.900E 01	



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## CONSTANT INJECTION RATE PROPOSED METHOD

TIME	OIL RECOVERY	PORE VOL.	INJ.	w.o.r.	PRESS. DROP
0.15139E 03	0.47969E-02	0.47969E-02	0.00	0.164E 01	
0.45152E 03	0.14307E-01	0.14307E-01	0.00	0.140E 01	
0.75125E 03	0.23804E-01	0.23804E-01	0.00	0.129E 01	
0.10541E 04	0.33401E-01	0.33401E-01	0.00	0.123E 01	
0.13585E 04	0.43042E-01	0.43042E-01	0.00	0.120E 01	
0.16632E 04	0.52700E-01	0.52700E-01	0.00	0.118E 01	
0.19683E 04	0.62365E-01	0.62365E-01	0.00	0.116E 01	
0.22735E 04	0.72038E-01	0.72038E-01	0.00	0.115E 01	
0.25790E 04	0.81716E-01	0.81716E-01	0.00	0.113E 01	
0.28846E 04	0.91400E-01	0.91400E-01	0.00	0.112E 01	
0.31903E 04	0.10109E 00	0.10109E 00	0.00	0.111E 01	
0.34961E 04	0.11077E 00	0.11077E 00	0.00	0.110E 01	
0.38018E 04	0.12046E 00	0.12046E 00	0.00	0.110E 01	
0.41074E 04	0.13014E 00	0.13014E 00	0.00	0.109E 01	
0.44130E 04	0.13983E 00	0.13983E 00	0.00	0.108E 01	
0.47185E 04	0.14951E 00	0.14951E 00	0.00	0.108E 01	
0.50240E 04	0.15919E 00	0.15919E 00	0.00	0.107E 01	
0.53293E 04	0.16886E 00	0.16886E 00	0.00	0.107E 01	
0.56345E 04	0.17853E 00	0.17853E 00	0.00	0.106E 01	
0.59396E 04	0.18820E 00	0.18820E 00	0.00	0.106E 01	
0.62446E 04	0.19786E 00	0.19786E 00	0.00	0.105E 01	
0.65493E 04	0.20752E 00	0.20752E 00	0.00	0.105E 01	
0.68539E 04	0.21717E 00	0.21717E 00	0.00	0.104E 01	
0.71581E 04	0.22681E 00	0.22681E 00	0.00	0.104E 01	
0.74621E 04	0.23644E 00	0.23644E 00	0.00	0.104E 01	
0.77658E 04	0.24606E 00	0.24606E 00	0.00	0.103E 01	
0.80691E 04	0.25567E 00	0.25567E 00	0.00	0.103E 01	
0.83721E 04	0.26527E 00	0.26527E 00	0.00	0.103E 01	
0.86746E 04	0.27486E 00	0.27486E 00	0.00	0.102E 01	
0.89766E 04	0.28443E 00	0.28443E 00	0.00	0.102E 01	
0.92780E 04	0.29398E 00	0.29398E 00	0.00	0.101E 01	
0.95789E 04	0.30351E 00	0.30351E 00	0.00	0.101E 01	
0.98790E 04	0.31302E 00	0.31302E 00	0.00	0.101E 01	
0.10178E 05	0.32251E 00	0.32251E 00	0.00	0.100E 01	
0.10476E 05	0.33196E 00	0.33196E 00	0.00	0.100E 01	
0.10773E 05	0.34137E 00	0.34137E 00	0.00	0.995E 00	
0.11069E 05	0.35072E 00	0.35072E 00	0.00	0.989E 00	
0.11362E 05	0.36001E 00	0.36001E 00	0.00	0.981E 00	
0.11651E 05	0.36918E 00	0.36918E 00	0.00	0.971E 00	
0.11920E 05	0.37771E 00	0.37771E 00	0.00	0.902E 00	

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## CONSTANT INJECTION RATE PROPOSED METHOD

TIME	OIL RECOVERY	PORE VOL.	INJ.	W.C.R.	PRESS. DROP
0.12154E 05	0.38402E 00	0.38494E 00	0.36	0.818E 00	
0.12370E 05	0.38862E 00	0.39160E 00	0.49	0.812E 00	
0.12585E 05	0.39314E 00	0.39839E 00	0.50	0.807E 00	
0.12799E 05	0.39765E 00	0.40518E 00	0.51	0.801E 00	
0.13012E 05	0.40211E 00	0.41194E 00	0.52	0.795E 00	
0.13224E 05	0.40653E 00	0.41867E 00	0.52	0.789E 00	
0.13436E 05	0.41091E 00	0.42538E 00	0.53	0.783E 00	
0.13647E 05	0.41526E 00	0.43207E 00	0.54	0.776E 00	
0.13860E 05	0.41967E 00	0.43883E 00	0.53	0.755E 00	
0.14080E 05	0.42417E 00	0.44639E 00	1.10	0.706E 00	
0.14312E 05	0.42852E 00	0.45431E 00	1.38	0.670E 00	
0.14549E 05	0.43114E 00	0.46180E 00	1.41	0.665E 00	
0.14785E 05	0.43419E 00	0.46924E 00	1.43	0.660E 00	
0.15020E 05	0.43725E 00	0.47672E 00	1.46	0.656E 00	
0.15256E 05	0.44027E 00	0.48418E 00	1.48	0.651E 00	
0.15490E 05	0.44327E 00	0.49163E 00	1.50	0.647E 00	
0.15724E 05	0.44623E 00	0.49907E 00	1.53	0.642E 00	
0.15958E 05	0.44912E 00	0.50646E 00	1.55	0.638E 00	
0.16191E 05	0.45202E 00	0.51387E 00	1.57	0.634E 00	
0.16424E 05	0.45488E 00	0.52126E 00	1.60	0.629E 00	
0.16657E 05	0.45771E 00	0.52864E 00	1.62	0.625E 00	
0.16889E 05	0.46050E 00	0.53600E 00	1.64	0.621E 00	
0.17121E 05	0.46322E 00	0.54330E 00	1.66	0.617E 00	
0.17353E 05	0.46597E 00	0.55056E 00	1.69	0.613E 00	
0.17584E 05	0.46873E 00	0.55803E 00	1.71	0.609E 00	
0.17815E 05	0.47142E 00	0.56534E 00	1.73	0.606E 00	
0.18046E 05	0.47465E 00	0.57264E 00	1.75	0.602E 00	
0.18276E 05	0.47671E 00	0.57994E 00	1.77	0.598E 00	
0.18506E 05	0.47936E 00	0.58727E 00	1.79	0.595E 00	
0.18736E 05	0.48197E 00	0.59457E 00	1.80	0.591E 00	
0.18966E 05	0.48455E 00	0.60184E 00	1.82	0.587E 00	
0.19196E 05	0.48714E 00	0.60914E 00	1.84	0.584E 00	
0.19426E 05	0.48973E 00	0.61645E 00	1.85	0.580E 00	
0.19656E 05	0.49228E 00	0.62375E 00	1.86	0.576E 00	
0.19886E 05	0.49482E 00	0.63103E 00	1.86	0.570E 00	
0.20119E 05	0.49743E 00	0.63840E 00	1.79	0.558E 00	
0.20357E 05	0.50018E 00	0.64593E 00	1.65	0.536E 00	
0.20603E 05	0.50267E 00	0.65379E 00	3.11	0.518E 00	
0.20853E 05	0.50454E 00	0.66180E 00	3.81	0.512E 00	
0.21103E 05	0.50617E 00	0.66972E 00	3.86	0.509E 00	

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TIME	CONSTANT OIL RECOVERY	INJECTION PORE VOL.	RATE INJ.	PROPOSED W.O.R.	METHOD PRESS. DROP
0.21354E 05	0.50776E 00	0.67763E 00	3.91	0.507E 00	
0.21604E 05	0.50932E 00	0.68552E 00	3.96	0.504E 00	
0.21855E 05	0.51093E 00	0.69348E 00	4.01	0.501E 00	
0.22105E 05	0.51250E 00	0.70141E 00	4.06	0.499E 00	
0.22356E 05	0.51409E 00	0.70937E 00	4.11	0.496E 00	
0.22606E 05	0.51563E 00	0.71730E 00	4.16	0.494E 00	
0.22856E 05	0.51714E 00	0.72522E 00	4.21	0.491E 00	
0.23107E 05	0.51871E 00	0.73321E 00	4.26	0.489E 00	
0.23357E 05	0.52018E 00	0.74112E 00	4.31	0.486E 00	
0.23608E 05	0.52167E 00	0.74905E 00	4.36	0.484E 00	
0.23858E 05	0.52316E 00	0.75700E 00	4.41	0.482E 00	
0.24108E 05	0.52462E 00	0.76493E 00	4.46	0.479E 00	
0.24358E 05	0.52609E 00	0.77289E 00	4.51	0.477E 00	
0.24609E 05	0.52751E 00	0.78081E 00	4.56	0.475E 00	
0.24859E 05	0.52894E 00	0.78876E 00	4.61	0.473E 00	
0.25110E 05	0.53042E 00	0.79678E 00	4.67	0.471E 00	
0.25361E 05	0.53181E 00	0.80472E 00	4.71	0.469E 00	
0.25612E 05	0.53316E 00	0.81263E 00	4.75	0.467E 00	
0.25863E 05	0.53457E 00	0.82062E 00	4.80	0.465E 00	
0.26114E 05	0.53593E 00	0.82857E 00	4.85	0.463E 00	
0.26365E 05	0.53730E 00	0.83654E 00	4.90	0.461E 00	
0.26616E 05	0.53863E 00	0.84449E 00	4.94	0.459E 00	
0.26867E 05	0.53996E 00	0.85245E 00	4.99	0.457E 00	
0.27119E 05	0.54134E 00	0.86047E 00	5.04	0.455E 00	
0.27370E 05	0.54263E 00	0.86840E 00	5.09	0.453E 00	
0.27621E 05	0.54392E 00	0.87635E 00	5.13	0.452E 00	
0.27872E 05	0.54522E 00	0.88432E 00	5.18	0.450E 00	
0.28124E 05	0.54652E 00	0.89231E 00	5.23	0.448E 00	
0.28375E 05	0.54783E 00	0.90031E 00	5.28	0.446E 00	
0.28627E 05	0.54904E 00	0.90822E 00	5.32	0.445E 00	
0.28878E 05	0.55025E 00	0.91615E 00	5.36	0.443E 00	
0.29130E 05	0.55157E 00	0.92419E 00	5.41	0.441E 00	
0.29381E 05	0.55284E 00	0.93219E 00	5.46	0.440E 00	
0.29633E 05	0.55406E 00	0.94015E 00	5.50	0.438E 00	
0.29884E 05	0.55529E 00	0.94813E 00	5.55	0.437E 00	
0.30136E 05	0.55652E 00	0.95612E 00	5.59	0.435E 00	
0.30388E 05	0.55775E 00	0.96413E 00	5.64	0.433E 00	
0.30640E 05	0.55894E 00	0.97210E 00	5.69	0.432E 00	
0.30891E 05	0.56013E 00	0.98009E 00	5.73	0.430E 00	
0.31143E 05	0.56125E 00	0.98801E 00	5.77	0.429E 00	

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TIME	OIL RECOVERY	PORE VOL.	INJ.	CONSTANT	INJECTION RATE	PROPOSED	METHOD
				w.o.r.	PRESS. DROP		
0.31395E 05	0.56245E 00	0.99600E 00	5.81	0.428E 00			
0.31647E 05	0.56365E 00	0.10040E 01	5.85	0.426E 00			
0.31899E 05	0.56486E 00	0.10120E 01	5.89	0.425E 00			
0.32150E 05	0.56606E 00	0.10201E 01	5.94	0.423E 00			
0.32402E 05	0.56715E 00	0.10280E 01	5.97	0.422E 00			
0.32654E 05	0.56831E 00	0.10360E 01	6.02	0.420E 00			
0.32905E 05	0.56947E 00	0.10440E 01	6.07	0.419E 00			
0.33157E 05	0.57063E 00	0.10520E 01	6.12	0.417E 00			
0.33409E 05	0.57173E 00	0.10599E 01	6.16	0.416E 00			
0.33660E 05	0.57278E 00	0.10679E 01	6.19	0.415E 00			
0.33912E 05	0.57395E 00	0.10759E 01	6.24	0.414E 00			
0.34163E 05	0.57513E 00	0.10839E 01	6.28	0.412E 00			
0.34415E 05	0.57619E 00	0.10919E 01	6.32	0.411E 00			
0.34666E 05	0.57718E 00	0.10998E 01	6.35	0.410E 00			
0.34918E 05	0.57830E 00	0.11078E 01	6.39	0.409E 00			
0.35170E 05	0.57942E 00	0.11158E 01	6.43	0.407E 00			
0.35421E 05	0.58049E 00	0.11238E 01	6.46	0.406E 00			
0.35673E 05	0.58156E 00	0.11318E 01	6.50	0.405E 00			
0.35925E 05	0.58257E 00	0.11397E 01	6.53	0.404E 00			
0.36177E 05	0.58358E 00	0.11476E 01	6.56	0.403E 00			
0.36429E 05	0.58466E 00	0.11556E 01	6.60	0.401E 00			
0.36681E 05	0.58575E 00	0.11637E 01	6.63	0.400E 00			
0.36923E 05	0.58677E 00	0.11716E 01	6.66	0.399E 00			
0.37186E 05	0.58779E 00	0.11796E 01	6.69	0.398E 00			
0.37438E 05	0.58889E 00	0.11876E 01	6.73	0.397E 00			
0.37690E 05	0.58999E 00	0.11957E 01	6.76	0.395E 00			
0.37943E 05	0.59103E 00	0.12037E 01	6.79	0.394E 00			
0.38195E 05	0.59201E 00	0.12117E 01	6.81	0.393E 00			
0.38448E 05	0.59300E 00	0.12196E 01	6.83	0.392E 00			
0.38700E 05	0.59404E 00	0.12277E 01	6.85	0.391E 00			
0.38953E 05	0.59518E 00	0.12358E 01	6.74	0.388E 00			
0.39207E 05	0.59629E 00	0.12439E 01	6.48	0.385E 00			
0.39463E 05	0.59738E 00	0.12520E 01	6.22	0.381E 00			
0.39720E 05	0.59852E 00	0.12601E 01	5.88	0.377E 00			
0.39980E 05	0.59954E 00	0.12684E 01	9.78	0.373E 00			
0.40242E 05	0.60038E 00	0.12768E 01	11.59	0.371E 00			
0.40504E 05	0.60110E 00	0.12852E 01	11.72	0.370E 00			
0.40767E 05	0.60179E 00	0.12935E 01	11.83	0.369E 00			
0.41029E 05	0.60240E 00	0.13018E 01	11.92	0.368E 00			
0.41292E 05	0.60299E 00	0.13101E 01	12.00	0.367E 00			

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TIME	CONSTANT OIL RECOVERY	INJECTION PORE VOL.	RATE INJ.	PROPOSED W.O.R.	METHOD PRESS. DROP
0.41554E 05	0.69366E 00	0.13184E 01	12.11	0.366E 00	
0.41949E 05	0.69470E 00	0.13310E 01	12.26	0.364E 00	
0.42738E 05	0.60659E 00	0.13561E 01	12.55	0.362E 00	
0.43926E 05	0.60906E 00	0.13934E 01	12.94	0.358E 00	
0.45385E 05	0.61191E 00	0.14393E 01	13.40	0.355E 00	
0.47255E 05	0.61554E 00	0.14981E 01	13.99	0.350E 00	
0.49679E 05	0.61991E 00	0.15743E 01	14.75	0.345E 00	
0.52672E 05	0.62495E 00	0.16684E 01	15.68	0.339E 00	
0.56254E 05	0.63043E 00	0.17808E 01	16.79	0.332E 00	
0.60310E 05	0.63621E 00	0.19081E 01	18.12	0.325E 00	
0.65004E 05	0.64248E 00	0.20557E 01	19.78	0.313E 00	
0.70501E 05	0.64902E 00	0.22284E 01	21.70	0.310E 00	
0.76831E 05	0.65581E 00	0.24273E 01	23.97	0.303E 00	
0.84176E 05	0.66285E 00	0.26583E 01	26.64	0.295E 00	
0.92570E 05	0.66997E 00	0.29222E 01	29.65	0.288E 00	
0.10205E 06	0.67709E 00	0.32203E 01	33.12	0.281E 00	
0.11279E 06	0.68412E 00	0.35534E 01	37.08	0.274E 00	









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